Library
Physics Library

REGISTERED NO. BG. 315

PROCEEDINGS

OF THE

INDIAN ACADEMY OF SCIENCES

VOL. LIII] *SECTION A* [No. 1

JANUARY 1961

Price Rs. 2 or 3 Sh. *Annual Subscription Rs. 18*

PRINTED AT THE BANGALORE PRESS, BANGALORE CITY, BY T. K. BALAKRISHNAN
SUPERINTENDENT, AND PUBLISHED BY THE INDIAN ACADEMY OF
SCIENCES, BANGALORE

SOME REMARKS ON THE STRUCTURE OF ELEMENTARY PARTICLE INTERACTIONS

BY ALLADI RAMAKRISHNAN, F.A.SC., A. P. BALACHANDRAN* AND N. R. RANGANATHAN**

(University of Madras, Madras, India)

Received July 25, 1960

1. INTRODUCTION

A FEW years ago, Gell-Mann (1956) suggested a minimal principle for electromagnetic interactions according to which the electromagnetic interactions between elementary particles can only be of the current-field type that occurs in quantum electrodynamics and cannot be purely of the magnetic moment type. In this paper we suggest two specific transformations, as additions to the now well-known mass reversal transformation, invariance under which will guarantee the minimal character of the electromagnetic interactions. Applications of these transformations to strong and weak interactions are considered. These considerations suggest a model of strong interactions and an isotopic spin scheme for leptons which are also discussed.

2. THE MASS GAUGE TRANSFORMATION

To understand the significance of this transformation, we notice that in quantum electrodynamics, the total mass of electrons minus the total mass of positrons is a constant of motion. This implies that the quantum electrodynamical Lagrangian is invariant under the following gauge transformation of the electron field:

$$\psi_e \to e^{-im_e}\psi_e; \quad \bar{\psi}_e \to \bar{\psi}_e e^{+im_e} \tag{1}$$

where m_e is the rest mass of the electron. If now we postulate that all the electromagnetic Lagrangians should be invariant under the transformations

$$\psi_A \to e^{-im_A}\psi_A; \quad \bar{\psi}_A \to \bar{\psi}_A e^{+im_A} \tag{2}$$

where ψ_A is the field and m_A the rest mass of any fermion, we obtain the minimal principle of Gell-Mann apart from the case where a particle is

* Research Fellow of the University of Madras.

** Senior Research Fellow of the Atomic Energy Commission.

coupled to itself by a magnetic moment type of interaction. The conservation law implied here is that the total mass of fermions minus the total mass of anti-fermions should be a constant of motion in the electromagnetic interactions. In the Feynman picture, the principle may be stated as follows. As we follow a fermion line in a Feynman diagram there is nothing to prevent a coupling that will make a fermion become another fermion of different mass at a vertex, the only conservation law imposed at a vertex being on energy and momentum. However in quantum electrodynamics we notice that as we follow a fermion line, its mass is conserved. If we now insist that this be true in all electromagnetic interactions we obtain the result stated above.

It is obvious that this invariance holds also for the strong pion-baryon interactions in the charge independence approximation where the mass differences within a multiplet are neglected, except for the $\Sigma - \Lambda - \pi$ coupling. In the global symmetry model [Gell-Mann (1957)] where the $\Sigma^0 - \Lambda^0$ mass difference can also be neglected in the absence of K-meson interactions, this symmetry holds for all the π-baryon couplings when the K-meson interactions are switched off.

Turning now to the weak interactions, decays like $\mu^{\pm} \rightarrow e^{\pm} + \nu + \bar{\nu}$ show that the fermion mass conservation is violated in weak decays. To deduce the universal character of the weak interactions from the violation of the fermion mass conservation, we suggest the following argument. The π-baryon interactions, in contrast to the electromagnetic interactions, allow a change of the electric charge of the fermions. These couplings also exhibit a higher degree of symmetry in that the couplings are the same for the different charge states of the same field. Pursuing this analogy, we may expect the weak interactions to involve a universal Fermi coupling (as is experimentally true) since these interactions allow the fermions to change their mass states. In this picture, as far as the weak interactions are concerned, all the fermion fields are different modes of a single fundamental fermion field.

As another application of the mass gauge transformation, we notice that the K-meson interactions connect fermions of different masses. Hence we may similarly expect the K-mesons to be symmetrically coupled to all the baryons. This is the 'cosmic symmetry' model of strong interactions arrived at by Sakurai (1959) from a different point of view. Also, apart from the $\Sigma - \Lambda - \pi$ coupling, the π-baryon couplings connect the same mass states of the fermions, neglecting of course the mass differences within a multiplet. This suggests that perhaps the $\Sigma - \Lambda - \pi$ coupling does not exist.

This has also been suggested by Feynman (1958). This will then ensure parity conservation for all the non-derivative pion interactions from CP invariance and charge independence in the conventional sense [Feinberg and Gursey (1959)]. The K-baryon interactions will also conserve parity because of their 'cosmic symmetry' [Sakurai (1959)].

3. The Mass Reversal Transformation†

Recently when studying the structure of Fermi interactions, Feynman and Gell-Mann (1958) showed that the Dirac equation can be written as a second order equation

$$\left(\gamma_\mu \frac{\partial}{\partial x_\mu} + m\right)\left(\gamma_\nu \frac{\partial}{\partial x_\nu} - m\right)\psi = 0. \tag{3}$$

If ψ is a solution of this new equation, $\psi' = \gamma_5\psi$ is also a solution. So is a linear combination of ψ and $\gamma_5\psi$. This is not true of the first order Dirac equation since γ_5 does not commute with the Dirac Hamiltonian. $\gamma_5\psi$ corresponds to the solution of the equation obtained by replacing m by $-m$ in the Dirac equation and this is equivalent to stating that ψ and $\gamma_5\psi$ are solutions of the Dirac equation of the second order. In considering the interactions of particles, Feynman and Gell-Mann suggested that these solutions of the second order equation should be treated as fundamental, of course imposing conditions depending upon the nature of the interactions. In the case of universal Fermi interactions, the linear combination of solutions used is $\frac{1}{2}(1 \pm \gamma_5)\psi$ which are the eigen states of the chiral operator γ_5 [Sudarshan and Marshak (1958)].

Since ψ and $\gamma_5\psi$ have opposite parities, we may insist that all the interaction Lagrangians be invariant under the simultaneous reversal of the parities of all the fermion fields by the transformation

$$\psi \rightarrow \psi' = \eta\gamma_5\psi \tag{4}$$

with $|\eta|^2 = 1$, since only the relative parities of fermions are physically significant [Wick, Wightman and Wigner (1950)]. This leaves the current-field type electromagnetic Lagrangian $ie\bar{\psi}\gamma_\mu\psi A_\mu$ invariant provided that $A_\mu \rightarrow A_\mu' = A_\mu$. If we accept this transformation for A_μ, then the magnetic moment type couplings are not invariant under these transformations and are consequently forbidden.

† This transformation has been discussed in various contents by Sakurai (1958), Okun (1958) and Salam (1959). We merely collect them here as a single connected series of results.

If we now require Yukawa couplings like $ig\bar{\psi}\gamma_5\vec{\tau}\psi\vec{\phi}$ to be invariant under this transformation, we find that $\vec{\phi}$ must transform as

$$\vec{\phi} \rightarrow \vec{\phi}' = -\vec{\phi}. \tag{5}$$

With this transformation for $\vec{\phi}$, it follows that there can be no derivative Yukawa couplings. Invariance under this transformation also forbids couplings between an odd number of spin zero bosons like $\lambda\phi^3$. [*see* also Baym (1960)]. These results are true whether ϕ is scalar or pseudoscalar.

4. THE τ_3 TRANSFORMATION

The minimal principle for the electromagnetic interactions of baryons may be formulated by specifying the form of the corresponding interaction Lagrangians in the isotopic spin space. If ψ_N is the nucleon field, its electromagnetic interaction can be written as $ie/2\,\bar{\psi}_N\gamma_\mu(1+\tau_3)\psi_N A_\mu$. Such an interaction is obviously not invariant under rotations about the first or second axis in the isotopic spin space and consequently it is not invariant under a reflection in the plane formed by the first and third axes of the isotopic spin space. Therefore if we define a parity for the fields for such a reflection, this parity is not conserved in electromagnetic interactions. This is analogous to what happens in weak interactions where space parity is not conserved. In the case of weak interactions, the universal $V \pm A$ interaction can be arrived at by arguing that the interaction should be unchanged even if the fields that occur in it are taken over to a state of opposite parity individually, as parity is not conserved. [Sakurai (1958), Sudarshan and Marshak (1958)]. This is achieved by the transformation $\psi \rightarrow \psi' = \pm\gamma_5\psi$. In a similar manner, we can expect the electromagnetic interactions to remain invariant under a transformation which switches the parity of a field under the reflection in the isospin space referred to above to the opposite parity. It is important that this transformation can be applied separately to each of the fields in the interaction. Such a transformation for isospinor fields would be

$$\psi \rightarrow \eta\tau_3\psi \tag{6}$$

with $|\eta|^2 = 1$. Then the electromagnetic coupling of fermions which are isospinors reduce to

$$\lambda\bar{\psi}_A O_i(1 \pm \tau_3)\psi_B f_i \tag{7}$$

where O_i and f_i are appropriate γ-matrices and functions of A_μ respectively. The two cases $(1 \pm \tau_3)$ belong to the cases where $\eta = \pm 1$. It is to be noted that couplings between fields of different η are forbidden so that both ψ_A and ψ_B should have the same η. In the d'Espagnat-Prentki scheme [d'Espagnat and Prentki (1956 and 1957)], fields with $\eta = +1$ correspond to isospinors of the first kind and those with $\eta = -1$ to isospinors of the second kind. When $\psi_A \equiv \psi_B$, the transformation has of course to be applied simultaneously to both $\bar{\psi}$ and ψ so that the form is not quite unique in that both $\lambda\bar{\psi}O_i\psi f_i$ and $\lambda\bar{\psi}O_i\tau_3\psi f_i$ are invariant under these transformations. However if we assume that (7) holds unchanged even in this case, we obtain the electromagnetic interaction of an isospinor with itself as $\lambda\bar{\psi}O_i(1 \pm \tau_3)\psi f_i$. For a spinor field which is an isopseudovector, the analogous transformation would be

$$\xi \rightarrow \xi' = \eta\theta_3\xi$$

where

$$\theta_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \qquad \xi = \begin{bmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{bmatrix} \tag{8}$$

with $|\eta|^2 = 1$. The coupling $\lambda\bar{\xi}O_i\theta_3\xi f_i$ is evidently invariant under (8). This form is also unique provided we assume the cylindrical symmetry of the electromagnetic interactions about the third axis in the isotopic spin space. This scheme is sufficient to ensure the minimal electromagnetic interaction of the baryons apart from the case where a particle is coupled to itself through a magnetic moment interaction. A coupling between Σ^+ and p or Ξ^- and Σ^- can be forbidden by choosing $\eta = \pm i$ for Σ even if we do not assume any rotational symmetry for electromagnetic interactions about the third axis in the isotopic spin space.

5. A Model of Strong Interactions

Our considerations regarding the mass gauge transformations have suggested a model of strong interactions according to which the K-meson interactions exhibit 'cosmic symmetry' in the sense of Sakurai [Sakurai (1959)] and the $\Sigma - \Lambda - \pi$ coupling is zero. The interaction Lagrangian in this model would be

$$L_{\text{int.}} = L_\pi + L_K$$

$$= iG_1\bar{N}_1\gamma_5\vec{\tau}N_1\vec{\pi} + iG_2\vec{\bar{\Sigma}}\times\vec{\Sigma}\gamma_5\vec{\pi} + iG_3\bar{N}_4\gamma_5\vec{\tau}N_4\vec{\pi}$$

$$+ F[(\bar{N}_1N_2)K^0 + (\bar{N}_1N_3)K^+ + (\bar{N}_4N_2)\bar{K}^+$$

$$- (\bar{N}_4N_3)\bar{K}^0 + h.c] \tag{9}$$

where, as usual

$$N_1 = \begin{pmatrix} p \\ n \end{pmatrix}, \quad N_2 = \begin{pmatrix} \Sigma^+ \\ Y^0 \end{pmatrix}$$

$$N_3 = \begin{pmatrix} Z^0 \\ \Sigma^- \end{pmatrix}, \quad N_4 = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix} \tag{10}$$

with

$$Y_0 = \frac{\Lambda^0 - \Sigma^0}{\sqrt{2}}; \quad Z^0 = \frac{\Lambda^0 + \Sigma^0}{\sqrt{2}}.$$

We also define

$$N_2' = \begin{pmatrix} \Sigma^+ \\ -Z^0 \end{pmatrix}; \quad N_3' = \begin{pmatrix} -Y^0 \\ \Sigma^- \end{pmatrix}; \tag{11}$$

which are obtained by replacing Λ^0 by $-\Lambda^0$ in N_2 and N_3.

In writing (9), we have assumed equal parity for all the baryons while the appropriate matrix 1 or $i\gamma_5$ has yet to be inserted in the K-baryon interactions depending upon the parity of the K-meson.

The Lagrangian (9) involves a totality of four different coupling constants and does not lead to any result independent of the perturbation theory in contradiction with experiments. In order to obtain a further reduction in the number of independent coupling constants, we may draw the following analogy between the K- and π-meson interactions and the π- and electromagnetic interactions. The π-meson interactions connect different charge states of a field and are known to be symmetric between the different charge states. The electromagnetic interactions break this symmetry. This breaking is not arbitrary, the couplings introduced for a triplet for instance being such that each member is coupled to itself with a coupling constant equal to eI_z. The electromagnetic interaction of an isotopic triplet thus transforms

as the third component of a vector in the isotopic spin space. Since in our model, the K-meson interactions which connect different hypercharge (U) states of the baryons are supposed to be symmetric and the π-meson interactions are supposed to break this symmetry, it is natural introduce the π-meson interactions in analogy with the electromagnetic interactions by setting $G_1 = -G_3$; $G_2 = 0$ since N_1 has $U = 1$, N_4 has $U = -1$ and N_2 and N_3 have $U = 0$. This implies that the π-interactions are cylindrically symmetrical about the third axis of the three-dimensional hypercharge space introduced by Feinberg and Gursey [Feinberg and Gursey (1959)]. The interaction Lagrangian now reduces to

$$\begin{aligned} L_{int.} = iG\,[\bar{N}_1\gamma_5\vec{\tau}N_1 - \bar{N}_4\gamma_5\vec{\tau}N_4]\,\vec{\pi} \\ + F\,[(\bar{N}_1N_2)\,K^0 + (\bar{N}_1N_3)\,K^+ + (\bar{N}_4N_2)\,\bar{K}^+ \\ - (\bar{N}_4N_3)\,\bar{K}^0 + h.c]. \end{aligned} \tag{12}$$

However this interaction is too highly symmetrical and as Pais has shown [Pais (1958)] leads to a number of results in contradiction with experiment. We may break this symmetry by insisting only on the equality of the absolute values of the K-meson coupling constants and setting for instance

$$f_{\Lambda N_1 K} = f_{\Sigma N_1 K} = f_{N_4 \Lambda K} = -f_{N_4 \Sigma K} = F. \tag{13}$$

We then obtain, using (11),

$$\begin{aligned} L_{int.} = iG\,[\bar{N}_1\vec{\tau}\gamma_5N_1 - \bar{N}_4\vec{\tau}\gamma_5N_4]\cdot\vec{\pi} \\ + F\,[(\bar{N}_1N_2)\,K^0 + (\bar{N}_1N_3)\,K^+ + (\bar{N}_4N_2')\,\bar{K}^+ \\ - (\bar{N}_4N_3')\,\bar{K}^0 + h.c). \end{aligned} \tag{14}$$

Considerations analogous to that of Pais [Pais (1958)] show that this removes the $\Sigma^0 - \Lambda^0$ degeneracy. However the invariance of (14) under

$$N_1 \to N_4,\ N_4 \to N_1,\ \vec{\pi} \to -\vec{\pi},\ \Lambda^0 \to -\Lambda^0,\ K^+ \to -\bar{K}^0,\ K^0 \to \bar{K}^+ \tag{15}$$

implies that the nucleon and the cascade are still degenerate. Invariance under (15) also implies certain equalities between the amplitudes involving the nucleon and the cascade which at present cannot be verified. It is interesting to note that the difficulty of introducing the mass difference

between N_1 and N_4 here is rather analogous to the difficulty of introducing the $\Sigma^+ - \Sigma^-$ mass difference which as pointed out by Gell-Mann [Gell-Mann (1957)] is rather too large to be explained as due to purely electromagnetic interactions.

The work of Pais [Pais (1958)] has already shown that such a small number of independent coupling constants as two is insufficient to reproduce all the known facts. It is however felt desirable to point out the suggestive analogy between the π- and electromagnetic interactions on the one hand and the K- and π-interactions on the other which does provide us with a definite way of introducing the π- and K-interactions. One can of course remove the degeneracy of the nucleon and the cascade by choosing different values for G_1 and G_3 still setting $G_2 = 0$ or by choosing $f_{\Lambda N_1 K} = f_{\Sigma N_1 K} \neq f_{\Lambda N_4 K} = -f_{\Sigma N_4 K}$. Such procedures however are rather arbitrary.

This model of course does not imply that the π-Σ-Σ or the π-Σ-Λ interactions are identically null. We always have virtual processes, where for instance the Σ dissociates into a nucleon and a K, the nucleon interacts with the π, and the resultant nucleon reabsorbs the K to become a Σ or a Λ. These are thus two-step processes and we may perhaps consequently expect the π-Σ and π-Λ interaction cross-sections to be smaller than the π-nucleon cross-sections.

We are grateful to Professor Abdus Salam for his comments and criticisms in the light of which the paper was revised. He has pointed out that as regards the mass gauge transformation, since we have made more use of the violation than the validity of the law of fermion mass conservation, the conclusions from them should not be pressed too far.

We also wish to thank Mr. K. Venkatesan for helpful discussions. One of us (N. R. R.) is indebted to the Atomic Energy Commission for the grant of a Senior Research Fellowship while the other (A. P. B.) is grateful to the University of Madras for the award of a Fellowship.

6. SUMMARY

The existence of symmetries which can lead to minimal electromagnetic interactions have been investigated and three specific invariances to obtain this minimal character have been proposed. Applications of these principles to strong and weak interactions are seen to lead to certain suggestive results. A model of strong interactions arising out of these considerations is also discussed.

REFERENCES

1. Gell-Mann, M. .. *Suppl. Nuovo Cim.*, 1955, **4,** 848.
2. ———— .. *Phys. Rev.*, 1957, **106,** 1296.
   Schwinger, J. .. *Ann. Phys.*, 1957, **2,** 407.
3. Sakurai, J. J. .. *Phys. Rev.*, 1959, **113,** 1679.
4. Feynman, R. P. .. *A Model of Strong and Weak Couplings* (Unpublished).
5. Feinberg, G. and Gursey, F. *Phys. Rev.*, 1959, **114,** 1153.
   See also Feinberg, G. .. *Ibid.*, 1957, **108,** 878.
   Gupta, S. N. .. *Can. J. Phys.*, 1957, **35,** 1309.
   Solov'ev, V. G. .. *J. Exptl. Theoret. Phys., U.S.S.R.*, 1957, **33,** 537, 796; translation, *Soviet Phys., J.E.T.P.*, 1957, **6** (33), 419, 613.
6. Feynman, R. P. and Gell-Mann, M. *Phys. Rev.*, 1958, **109,** 193.
7. Sakurai, J. J. .. *Nuovo Cim.*, 1958, **7,** 649.
8. Okun', L. B. .. *Proceedings of the* 1958 *Annual International Conference on High Energy Physics at CERN*, pp. 223.
9. Salam, A. .. *Phys. Rev. Lett.*, 1959, **2,** 127.
10. Sudarshan, E. C. G. and Marshak, R. E. *Phys. Rev.*, 1958, **109,** 193.
11. Wick, G. C., Wightman, A. and Wigner, E. P. *Ibid.*, 1950, **88,** 101.
12. Baym, G. .. *Phys. Rev.*, 1960, **117,** 886.
13. d'Espagnat, B. and Prentki, J. .. *Nucl. Phys.*, 1956, **1,** 33 and 1957, **3,** 446.
14. Pais, A. .. *Phys. Rev.*, 1958, **110,** 574.
15. Lee T. D. and Yang, C. N. "Implications of the Intermediate Boson Basis of the Weak Interactions: The Existence of a Quartet of Intermediate Bosons and Their Dual Isotopic Spin Transformation Properties," *Institute for Advanced Study* (Preprint, 1960).
16. Okun, L. B. and Pontecorvo, B. M. *J. Exptl. Theoret. Phys.*, 1958, **34,** 763.

APPENDIX

The analogy between the τ_3- and γ_5- transformations discussed in Section 4 leads to an isotopic spin scheme for leptons in which μ^- and e^- are one of the components of doublets whose other components are missing and ν is a singlet. This scheme is discussed below and it is found that it is able to explain many of the features of leptonic interactions in a qualitative way.

THE ISOTOPIC SPIN SCHEME FOR LEPTONS

In Section 4, we have discussed the fact that the electromagnetic interactions of baryons are invariant under the transformation $\psi \rightarrow \eta\tau_3\psi$ with $|\eta|^2 = 1$. This is analogous to the γ_5- transformation of the neutrino field and eliminates one of the components of the baryon field in its interaction with the electromagnetic field. We now extend the above transformation to μ and e fields and assume that they possess the following invariance similar to the γ_5-invariance of the neutrino field:

$$\mu = -\tau_3\mu$$
$$e = -\tau_3 e \tag{A1}$$

where the μ and e fields are taken to form isodoublets. It follows that μ and e are essentially one component isotopic spinors of the form

$$\mu = \begin{bmatrix} 0 \\ \mu^- \end{bmatrix}, \quad e = \begin{bmatrix} 0 \\ e^- \end{bmatrix}. \tag{A2}$$

We further assume that the neutrino is an isosinglet.

STRONG INTERACTIONS

It will now be shown that with the isotopic spin assignments given above, it is possible to forbid the strong interactions of leptons. We assume

(*a*) Strong interactions conserve the total isotopic spin. It is immediately clear that μ^- and e^- can have no strong interactions since no isotopic scalar can be formed involving μ or e because of their missing components.

If we further assume

(*b*) Strong interactions conserve parity, ν also can have no strong interactions since it always occurs in the combination $\frac{1}{2}(1+\gamma_5)\nu$.

It follows that leptons can have no strong interactions.

In electromagnetic interactions, the combination $\bar{\mu}\mu^-$ or $\bar{e}e^-$ occurs which transforms like a scalar plus the third component of a vector which is the correct transformation of these interactions for isospinors in the isotopic spin space.

It is interesting to note that only the leptons seem to possess invariances under transformations which eliminate one of their components, either in Lorentz space (γ_5-invariance of the neutrino) or in isotopic spin space ($-\tau_3$-invariance of μ and e) while for baryons, such invariances seem to be a property only of some of their interactions and not of the fields themselves.

Weak Interactions

Present evidence in weak interactions seems to rule out the existence of neutral lepton currents [Lee and Yang (1960)]. We therefore assume that the only leptonic currents relevant for weak interactions are $(\bar{\mu}\nu)$ and $(\bar{e}\nu)$. These transform as one of the components of an isospinor. We now assume that weak interactions involving leptons in which baryons are also involved obey the rule $|\vec{\Delta I}| = 1/2$ or $3/2$ while weak interactions involving only leptons obey $|\vec{\Delta I}| = 0$ or 1. Strangeness-changing baryonic currents satisfying $|\Delta S| = 1$ necessarily transform as isospinors corresponding to $I = 1/2$ or $3/2$. Consequently an interaction of the form $J_a^{SNC^+} J_a^l$ with $|\Delta S| = 1$ will not obey the selection rule $|\vec{\Delta I}| = 1/2$ or $3/2$ and are consequently forbidden. Thus we get the result that strangeness-changing leptonic decays with $|\Delta S| = 1$ are forbidden. Experimentally, apart from the $K \to \mu + \nu$ decay, all such decays have low probabilities. The $K \to \mu + \nu$ decay is hard to explain. Thus apart from this decay mode, we are able to explain at least qualitatively the low rates of such processes.

In this scheme, if $|\Delta S| = 2$ currents are allowed, we may have decays like $\Xi^- \to n + e^- + \bar{\nu}$. The $K_1^0 - K_2^0$ mass difference however seems to suggest against the existence of such currents [Okun and Pontecorvo (1958)].

ON COMPRESSIBLE BOUNDARY LAYER ON A FLAT PLATE WITH UNIFORM SUCTION

By A. C. Jain

(Department of Applied Mathematics, Indian Institute of Science, Bangalore-12)

Received August 19, 1960

(Communicated by Prof. P. L. Bhatnagar, F.A.SC.)

Summary

A quartic profile in terms of the normal distance from the wall has been taken and coefficients are evaluated by satisfying one more boundary condition on the wall than the usual one. By doing so, the limitations about the Reynolds number of the quartic profile adopted by Lew (1949) has been removed. The Kármán (1921) Momentum Integral Equation has been used to evaluate the various characteristics of the flow. A comparative study of Lew's quartic profile and exponential profile together with the quartic profile of the present paper has been undertaken and the graphs for the various characteristics of the flow for a number of Mach numbers and suction coefficients have been drawn. At the end, certain conclusions of general nature about the velocity profiles have been recorded.

1. Introduction

Lew (1949) solved the problem of boundary layer control for Prandtl number unity with specific heat C_p at constant pressure as constant and viscosity temperature relation given by $\mu \infty T^n$. He took a quartic profile as an approximation to the actual profile in the boundary layer and found that his results are applicable up to the Reynolds number 4×10^5 only. To overcome this difficulty, he used an exponential profile, which apart from other unsatisfactory features, gives substantially higher values of the characteristics of the flow throughout the entire range of Reynolds numbers.

In what follows, an attempt has been made to improve his solution so that the results are valid throughout the entire range of Reynolds numbers. This has been made possible by taking a quartic profile in terms of the distances normal to the wall and evaluating the coefficients by satisfying one more boundary condition on the wall than the usual ones.

A comparative study of the present quartic profile with Lew's quartic profile along with his exponential profile has been made and the graphs for

the various characteristics of the flow for a number of Mach numbers and suction coefficients have been drawn. At the end, certain conclusions of general nature about the suitability of the various profiles have been recorded.

Assumptions.—The analysis is based on the following assumptions:

(i) $\frac{\partial}{\partial x} \ll \frac{\partial}{\partial y}$,

(ii) Prandtl number σ is unity,

(iii) $\left(\frac{\partial T}{\partial y}\right)_0 = 0$, where the suffix '0' denotes the value on the plate.

(iv) c_p = constant

(v) $\mu \propto T^n$

(vi) $\rho\mu$ = constant.

Assumption (i) is the usual boundary layer approximation. Prandtl number $\sigma = (\mu c_p/k) = 1$ is taken in order to obtain a particular solution of the energy equation in the case of no heat transfer. Actual value of σ for air is 0·74 and, therefore, it slightly overestimates the viscosity. The assumption c_p = constant holds for a wide range of temperature. As for the assumption (v), n varies from 0·768 to 0·5 depending on the temperature range. At room temperature, n is approximately 0·768 but it approaches to 0·5 at higher temperatures. In this paper, all the numerical work has been carried out for $n = 0{\cdot}768$. $\rho\mu$ = constant is taken only to make the velocity profile independent of temperature distribution.

1. Equations of Motion

The equations determining a steady compressible laminar boundary layer flow on the plate without pressure gradient are the equation of momentum

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right), \tag{1.1}$$

the equation of continuity

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0, \tag{1.2}$$

the equation of energy

$$\rho u \frac{\partial}{\partial x}(c_p T) + \rho v \frac{\partial}{\partial y}(c_p T) = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \mu \left(\frac{\partial u}{\partial y}\right)^2, \tag{1.3}$$

and the equation of state

$$p = R\rho T. \tag{1.4}$$

In the above equations x, y are the distances measured along and perpendicular to the length of the plate and u, v are the velocity components in the corresponding directions. Other symbols have their usual meanings.

These equations are to be solved under the following boundary conditions:

$$\left.\begin{aligned} &\text{(i) At } y = 0,\ u = 0,\ v = v_0 = \text{constant } (v_0 < 0 \text{ for suction} \\ &\qquad \text{and } v_0 > 0 \text{ for injection}), \\ &\text{and} \\ &\text{(ii) At } y = \delta(x),\ u = U,\ T = T_1, \end{aligned}\right\} \tag{1.5}$$

where $\delta(x)$ is the boundary layer thickness at any section x and T_1 is the temperature in the main flow.

Multiplying (1.2) by u and adding to (1.3), we get

$$\rho u \frac{\partial T^x}{\partial x} + \rho v \frac{\partial T^x}{\partial y} = \left(\frac{1}{\sigma} - 1\right) \frac{\partial}{\partial y}\left(\mu \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial T^x}{\partial y}\right), \tag{1.6}$$

where

$$T^x = T + \frac{1}{2c_p} u^2$$

when

$$\sigma = 1,$$

$$T^x = \text{constant} \tag{1.7}$$

is thus always a particular solution of the equation (1.6), regardless of the distribution of pressure at the free-stream boundary. Since

$$\frac{\partial T}{\partial y} = -\frac{u}{c_p} \frac{\partial u}{\partial y}$$

vanishes at the wall, the integral (1.7) solves the problem of no heat transfer for any given outer velocity distribution. We note that it implies

$$T + \frac{1}{2c_p} u^2 = T_w = \text{constant}.$$

This particular integral (1.7) makes the analysis much simpler as once the velocity distribution is known, the corresponding temperature distribution can be easily calculated from (1.7).

The differential equations, governing the variation of u, v, ρ and T are (1.1), (1.2) and (1.6) together with the perfect gas relation (1.4). These equations are to be solved under the boundary conditions (1.5). Since, the pressure in the boundary layer is supposed to be known and T is given by (1.7) in case u is known, therefore, ρ is known from (1.4). As such the solution of these equations reduce to finding the velocity components u and v from equations (1.1) and (1.2) with the usual boundary conditions (1.5).

2. Solution of the Equations

As a first step towards the solution of the equations (1.1) and (1.2), we transform the co-ordinate system from (x, y) to (x, t) plane by the following Dorodnitzyn (1942) transformation:

$$x = x,$$

$$t = \int_0^y \frac{\rho}{\rho_\omega}\, dy. \tag{2.1}$$

The transformed momentum equation in the new variables with the assumption (vi) becomes [Kuerti (1951)]

$$u\frac{\partial u}{\partial x} + \bar{v}\frac{\partial u}{\partial y} = \nu_\omega \frac{\partial^2 u}{\partial t^2}, \tag{2.2}$$

where

$$\bar{v} = u\frac{\partial t}{\partial x} + \frac{\rho}{\rho_\omega} v, \tag{2.3}$$

and the equation of continuity (1.2) is

$$\frac{\partial u}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0. \tag{2.4}$$

The boundary conditions (1.5) take the form

(i) At $t = 0$, $u = 0$, $v = v_0 = \text{constant}$

(ii) At $t = \delta_t$, $u = U$ (2.5)

where

$$\delta(x) = \int_0^{\delta_t} \frac{\rho_\omega}{\rho}\, dt,$$

$$= \int_0^{\delta_t} \left(1 - \frac{u^2}{2c_p T_\omega}\right) dt. \tag{2.6}$$

It is to be noticed that equations (2.2) and (2.4) are of the same form as the corresponding equations in the incompressible fluid, the effect of the compressibility is confined only in the transformation of the normal co-ordinates and normal component of the velocity in the boundary layer.

Variables are made dimensionless with corresponding quantities at the wall in the following manner:

$$\left.\begin{aligned} &\bar{u} = \frac{u}{\sqrt{2c_p T_\omega}}, \qquad \bar{\bar{v}} = \frac{v}{\sqrt{2c_p T_\omega}}, \qquad \bar{\rho} = \frac{\rho}{\rho_\omega}, \\ &\bar{T} = \frac{T}{T_\omega}, \qquad \bar{\mu} = \frac{\mu}{\mu_\omega}, \qquad \bar{t} = \frac{t}{L}, \qquad \bar{x} = \frac{x}{L}, \end{aligned}\right\} \tag{2.7}$$

where L is some characteristic length.

Equations (2.2) and (2.4) in the dimensionless form become

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{\bar{v}}\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{1}{R_e}\frac{\partial^2 \bar{u}}{\partial \bar{t}^2}, \tag{2.8}$$

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{\bar{v}}}{\partial \bar{t}} = 0, \tag{2.8 a}$$

where

$$R_e = \frac{\rho_\omega L\sqrt{2c_p T_\omega}}{\mu_\omega}$$

and the boundary conditions (2.5) reduce to

$$\bar{t} = 0, \qquad \bar{u} = 0, \qquad \bar{\bar{v}} = \bar{v}_0$$

where

$$\bar{t} = \bar{\delta}_t, \qquad \bar{u} = \bar{U}, \qquad v_0 = \frac{v_0}{\sqrt{2c_p T_\omega}} \tag{2.9}$$

Integrating the boundary layer equation (2.8) across the boundary layer from 0 to $\bar{\delta}_t$ and using (2.8*a*) to eliminate $\bar{\bar{v}}$, we get the well-known Von Kármán momentum integral equation as:

$$\frac{\partial}{\partial s}\int_0^{\bar{\delta}_t} \bar{u}\,(\bar{U}-\bar{u})\,d\bar{t} - \mathrm{R}_e\bar{v}_0\bar{u} = \left(\frac{\partial\bar{u}}{\partial\bar{t}}\right)_0 \tag{2.10}$$

where

$$s = \mathrm{R}_e\bar{x}.$$

Since, in the boundary layer problem, we are concerned with the calculation of skin-friction, which mainly depends on the conditions near the body, we should place more emphasis on the boundary conditions near the body rather than at the outer edge of the boundary layer. We obtain the requisite number of boundary conditions at the body by differentiating the boundary layer equation and continuity equation [Howarth (1934)]. By doing so, the difficulty experienced by Lew in the case of his quartic profile is eliminated. These conditions have been first used by Torda (1951) for the incompressible case.

We assume the following form of the velocity profile:

$$\frac{\bar{u}}{\bar{U}} = a(s)\,\tau + b(s)\,\tau^2 + c(s)\,\tau^3 + d(s)\,\tau^4, \tag{2.11}$$

where

$$\tau = \frac{\bar{t}}{\bar{\delta}_t}.$$

The boundary conditions

$$\left.\begin{array}{ll} & \left\{\begin{array}{l} \bar{u} = 0, \\ \bar{v}_0\left(\dfrac{\partial\bar{u}}{\partial\bar{t}}\right)_0 = \dfrac{1}{\mathrm{R}_e}\left(\dfrac{\partial^2\bar{u}}{\partial\bar{t}^2}\right)_0, \\ \bar{v}_0\left(\dfrac{\partial^2\bar{u}}{\partial\bar{t}^2}\right)_0 = \dfrac{1}{\mathrm{R}_e}\left(\dfrac{\partial^3\bar{u}}{\partial\bar{t}^3}\right)_0, \end{array}\right. \\ \text{at} \quad \tau = 0 & \\ \text{and} & \\ \text{at} \quad \tau = 1 & \left\{\begin{array}{l} \bar{u} = \bar{U}, \\ \dfrac{\partial\bar{u}}{\partial\bar{t}} = 0, \end{array}\right. \end{array}\right\} \tag{2.12}$$

give the following values of the coefficients:

$$\left.\begin{aligned} a(s) &= \frac{24}{M^2+6M+18}, \qquad b(s) = \frac{12M}{M^2+6M+18}, \\ c(s) &= \frac{4M^2}{M^2+6M+18}, \qquad d(s) = \frac{-3(M^2+2M+2)}{M^2+6M+18}, \end{aligned}\right\} \tag{2.13}$$

where

$$M = R_e \bar{v}_0 \bar{\delta}_t.$$

It is clear that for no real value of M, the coefficients become infinite.

Substituting the expression for the velocity profile (2.11) with the coefficients (2.13) into the Von Kármán Integral equation (2.10), we get

$$\frac{R_e^2 v_0^2}{U}\, ds = \frac{4}{35}\, \frac{M\,(M^6+18M^5+157M^4+914M^3+3588M^2+7572M+7308)}{(M^2+6M+18)^2\,(M^3+6M^2+18M+24)}. \tag{2.14}$$

Integrating (2.14) between the limits 0 to M and evaluating the constant of integration from the condition:

$$M = 0 \quad \text{at} \quad s = 0.$$

we get

$$\begin{aligned} \xi = \left(\frac{v_0}{U}\right)^2 \left(\frac{Ux}{v_1}\right) = \frac{4}{35} \Big\{1 + \frac{\gamma-1}{2} M_\infty^2\Big\}^{n+1} \\ \times \Big\{ M + \frac{4M^2+99M}{3(M^2+6M+18)} \\ - 3 \log \frac{M^2+6M+18}{18} - 5{\cdot}0848 \log \frac{M+2{\cdot}626}{2{\cdot}626} \\ + 5{\cdot}5424 \log \frac{M^2+3{\cdot}374\,M+9{\cdot}1399}{9{\cdot}1399} \\ + 17 \tan^{-1} \frac{M}{M+6} \\ - 17.4 \tan^{-1} \frac{2{\cdot}5088}{1{\cdot}687M+9{\cdot}1399} \Big\}, \end{aligned} \tag{2.15}$$

where

$$M_\infty = \frac{U}{a}, \; a \text{ being the velocity of sound in the main flow.}$$

The equation (2.15) establishes the relation between the parameter M and the actual length of the plate ξ. The frictional drag coefficient, velocity profiles and boundary layer thickness can now be calculated in terms of the parameter M and related to the actual run of length ξ through equation (2.15).

The coefficient of average friction is defined by

$$C_F = \frac{\int_0^x \left(\mu \frac{\partial u}{\partial y}\right)_0 dx}{\frac{1}{2}\rho_1 U^2 x} \tag{2.16}$$

$$= \frac{1}{(R_{ex})\left(\frac{v_0}{U}\right)} \left\{1 + \frac{\gamma - 1}{2} M_\infty^2\right\}^n F(M)$$

$$- 2\left(\frac{v_0}{U}\right) \frac{1}{1 + \frac{\gamma - 1}{2} M_\infty^2}, \tag{2.17}$$

where

$$F(M) = \frac{8}{35} \cdot \frac{M(M^4 + 13M^3 + 89M^2 + 278M + 406)}{(M^2 + 6M + 18)^2}$$

and

$$R_{ex} = \frac{Ux}{\nu_1}.$$

The relation between the transformed ordinate t and the physical ordinate y can be established with the help of the relation (2.1), *viz.*,

$$y = \delta_t L \int_0^\tau (1 - \bar{u}^2)\, d\tau. \tag{2.18}$$

Using (2.11) with (2.13) in (2.18), we get

$$-\frac{v_0 y}{\nu_1} = -\left\{1 + \frac{\gamma - 1}{2} M_\infty^2\right\}^{n+1}$$

$$\times M\left\{\tau - \frac{\gamma - 1}{2} M_\infty^2 \frac{G(M, \tau)}{1 + \frac{\gamma - 1}{2} M_\infty^2}\right\}, \tag{2.19}$$

where

$$G(M,\tau) = \frac{\tau^3}{(M^2+6M+18)^2}\left\{192 + 144M\tau + \frac{336}{5}M^2\tau^2 + 8\tau^3[2M^3 - 3(M^2+2M+2)] + 8/7\, M\tau^4[2M^3 - 9(M^2+2M+2)] - 3M^2\tau^5(M^2+2M+2) + \tau^6(M^2+2M+2)^2\right\}.$$

Putting $\tau = 1$ in (2.19), we get the following equation giving us the boundary layer thickness in terms of the parameter M:

$$-\frac{v_0\delta}{\nu_1} = -\left\{1+\frac{\gamma-1}{2}M_\infty{}^2\right\}^n \times M\left\{1+\frac{\gamma-1}{2}M_\infty{}^2[1-G(M,1)]\right\}, \tag{2.20}$$

where

$$G(M,1) = \frac{2}{35}\frac{1}{(M^2+6M+18)^2}(5M^4+65M^3+431M^2 + 1460M + 2590).$$

Lastly, the velocity profile, given by (2.11) and the temperature distribution, given by (1.7) are related through equation (2.15) and (2.19) for any Mach number M_∞ and suction coefficient v_0/U.

3. For sake of comparison, we record below the $\xi - \lambda$ relation, C_F, $y - \tau$ relation and thickness of the boundary layer as obtained by Lew* where his λ is equal to $\frac{1}{2}M$ of the present paper

$$\xi = \frac{2}{35}\left(1+\frac{\gamma-1}{2}M_\infty{}^2\right)^{n+1}\left\{4\lambda - \frac{30}{\lambda+3} + 10 + \frac{9}{2}\log\frac{(\lambda+3)^2}{3(\lambda^2+3\lambda+3)} - \frac{35}{\sqrt{3}}\tan^{-1}\frac{\lambda}{\sqrt{3}(\lambda+2)}\right\}. \tag{3.1}$$

* The expressions (3.1), (3.2), (3.4), (3.7) and (3.8) in Lew's paper contain some misprints. For example, the log and arc tan terms in (3.1) are wrongly printed as

$$\log\frac{(\lambda+3)^2}{3(\lambda^2+3\lambda+3)},$$

$$arc\ \tan\{\lambda\sqrt{3}(\lambda+2)\}.$$

$$C_F = \frac{1}{(R_{ex})\left(\frac{v_0}{U}\right)}\left(1+\frac{\gamma-1}{2}M_\infty{}^2\right)^n f(\lambda)$$

$$-2\left(\frac{v_0}{U}\right)\frac{1}{1+\frac{\gamma-1}{2}M_\infty{}^2}, \tag{3.2}$$

where

$$f(\lambda) = \frac{4}{35}\times\frac{\lambda(4\lambda^2+26\lambda+37)}{(\lambda+3)^2}.$$

Also,

$$-\frac{v_0 y}{\nu_1} = -2\left(2+\frac{\gamma-1}{2}M_\infty{}^2\right)^{n+1}$$

$$\times\lambda\left\{\tau-\frac{\gamma-1}{2}M_\infty{}^2\frac{G(\lambda,\tau)}{1+\frac{\gamma-1}{2}M_\infty{}^2}\right\}, \tag{3.3}$$

where

$$G(\lambda,\tau) = \frac{\tau^3}{70(\lambda+3)^2}[840+1260\lambda\tau+168(3\lambda^2-8\lambda-6)\tau^2$$

$$-140(8\lambda^2+3\lambda-3)\tau^3+40(25\lambda^2+33\lambda+9)\tau^4$$

$$-105(4\lambda^2+7\lambda+3)\tau^5+70(\lambda+1)^2\tau^6],$$

and

$$-\frac{v_0\delta(x)}{\nu_1} = -2\lambda\left\{1+\frac{\gamma-1}{2}M_\infty{}^2\right\}^n$$

$$\times\left\{1+\frac{\gamma-1}{2}M_\infty{}^2[1-g(\lambda)]\right\}, \tag{3.4}$$

where

$$g(\lambda) = \frac{34\lambda^2+221\lambda+367}{70(\lambda+3)^2}.$$

Lew's velocity profile gives values of $u/U > 1$ for values of $\lambda < -1{\cdot}5$ so much so that when $\lambda = -3$ the velocity becomes infinite. The graphs for $U\delta/\nu_1$ against ξ shows abrupt changes when $\lambda < -1{\cdot}5$ showing thereby that his expression for boundary layer thickness does not hold for values of $\lambda < -1{\cdot}5$. We can see that this corresponds to Reynolds number Ux/ν_1

$= 4 \times 10^5$. Hence, the above results are applicable only for Reynolds number $< 4 \times 10^5$.

Lew overcame this difficulty by choosing the following exponential form of the velocity profile

$$\frac{\bar{u}}{\bar{U}} = 1 - e^{-\tau}\,[1 - \tau k\,(s)], \tag{3.5}$$

where $k\,(s)$ is so chosen as to satisfy boundary condition

$$\bar{v}_0 \left(\frac{\partial \bar{u}}{\partial \bar{t}}\right)_0 = \frac{1}{R_e}\left(\frac{\partial^2 \bar{u}}{\partial \bar{t}^2}\right)_0$$

on the plate, so that

$$k\,(s) = -\frac{1}{2}\,\frac{1+2\lambda}{1+\lambda}\,. \tag{3.6}$$

It is clear from (3.8) that this velocity profile satisfies the boundary conditions

$$\frac{\bar{u}}{\bar{U}} = 1, \quad \frac{\partial \bar{u}}{\partial \tau} = 0, \quad \frac{\partial^2 \bar{u}}{\partial \tau^2} = 0, \quad \frac{\partial^3 \bar{u}}{\partial \tau^3} = 0, \quad \ldots$$

at $\tau = \infty$ and not at $\tau = 1$.

For exponential velocity profile, Lew finds that

$$\xi = \left(\frac{v_0}{\bar{U}}\right)^2 \left(\frac{Ux}{\nu_1}\right) = \frac{1}{2}\left[1 + \frac{\gamma - 1}{2}\,M_\infty{}^2\right]^{n+1}$$

$$\times \left[\frac{\lambda\,(3\lambda+5)}{\lambda+1} + 5 \log \frac{1+\lambda}{1+2\lambda}\right], \tag{3.7}$$

and

$$C_F = \frac{1}{(R_{ex})\left(\frac{v_0}{\bar{U}}\right)} \left\{1 + \frac{\gamma - 1}{2}\,M_\infty{}^2\right\}^n \times \frac{\lambda\,(12\lambda^2 + 24\lambda + 11)}{4\,(1+\lambda)^2}$$

$$- 2\left(\frac{v_0}{\bar{U}}\right) \frac{1}{1 + \frac{\gamma - 1}{2}\,M_\infty{}^2}\,. \tag{3.8}$$

4. Discussion of the Results

In Fig. 1, the velocity profiles for $M_\infty = 1$ and 3 have been drawn. It is shown that the effect of Mach number on the velocity profile is similar

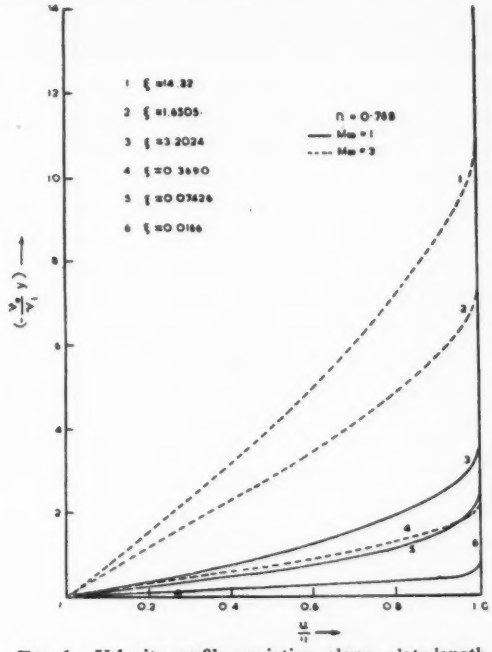


FIG. 1. Velocity profiles variation along plate length.

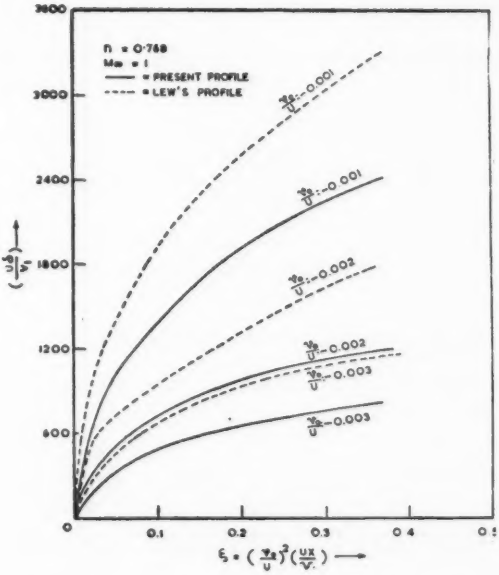


FIG. 2. Boundary layer thickness variation with suction coefficient

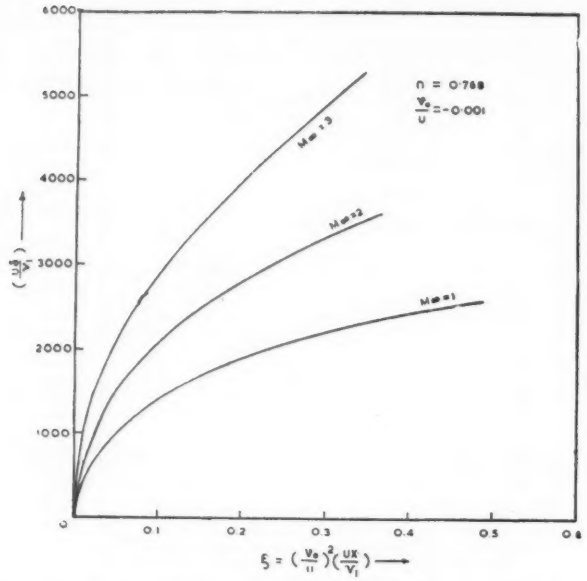


FIG. 3. Boundary layer thickness variation with Machnumber along plate length.

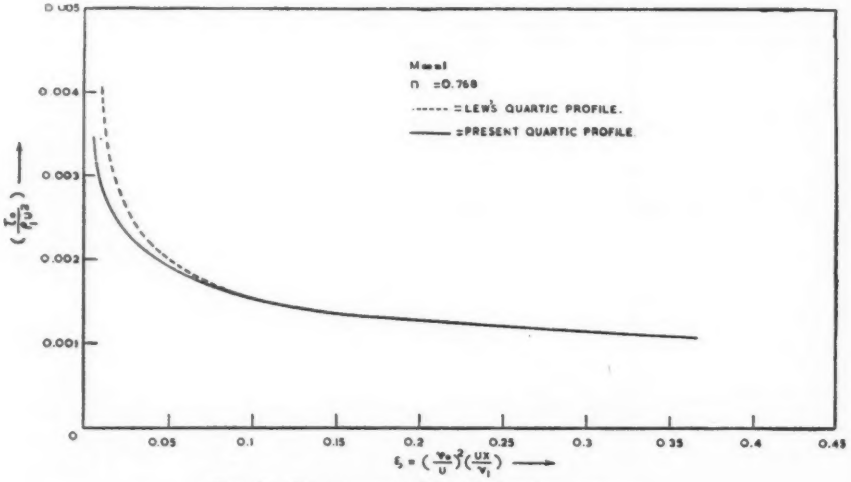


FIG. 4. Comparison of local shearing stresses

to the effect of a favourable pressure gradient along the length of the plate. With the increase of Mach number, the profiles become nearly linear [Karman and Tsien (1948)]. In Fig. 2, graphs for the boundary layer thickness against the length of the plate ξ for $M_\infty = 1$ and the suction coefficients $v_0/U = -\cdot001$, $-\cdot002$ and $-\cdot003$ have been drawn. It is found that boundary layer thickness decreases markedly as the suction coefficient increases and the present profile gives lower values of the boundary layer thickness than the Lew's quartic profile. In Fig. 3, it is shown that as Mach number increases, the boundary layer thickness increases. In Fig. 4, local shearing stresses as given by the quartic profile of Lew and of the present paper have been compared and it is found that the present profile gives lesser values of the local shearing stresses for $M_\infty = 1$ and $(v_0/U) = -\cdot001$ for Reynolds number up to $8\cdot5 \times 10^4$. Beyond this range, the two curves are the same in the range of validity of Lew's profile. In Fig. 5, various curves for the drag coefficient of the present profile for $M_\infty = 1$ and various values of the suction coefficients have been drawn and it is

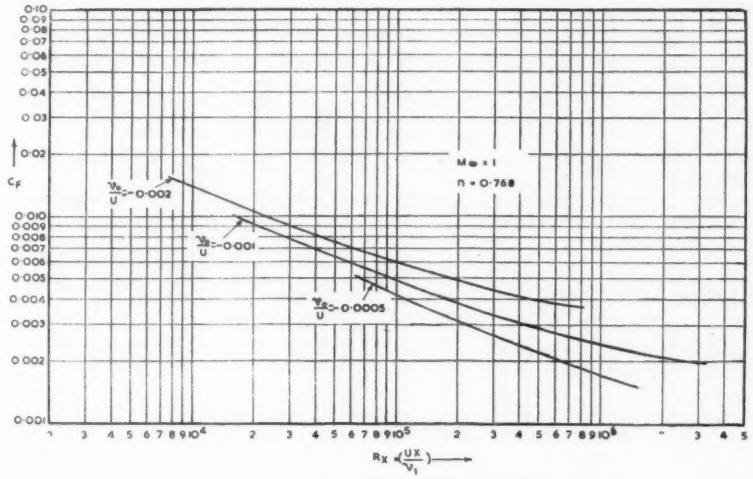


FIG. 5. Variation of friction coefficient with suction.

found that, as the suction coefficient increases the drag coefficient also increases. This is expected as due to suction the boundary layer thickness decreases markedly as shown in Fig. 2, therefore, the curve for the velocity profile will rise steeply to gain the value of the velocity in the outer flow. As such, $\tau_0 = \mu_\omega (\partial u/\partial y)_\omega$ will increase and therefore, the drag coefficient will increase. However, due to the thinning of the boundary layer

there will be increased stability and the velocity profiles will be less prone to transition. It is, therefore, expected that the net drag of the body will be reduced significantly due mainly to delayed transition.

In Fig. 6, it is shown that as the Mach number increases, the drag coefficient decreases. Also, a comparative study of the drag coefficients of the present profile with Lew's quartic profile and exponential profile for $M_\infty = 0$, 1 and $(v_0/U) = -\cdot 001$ has been made. It is found that in the range of applicability of Lew's quartic profile, the present profile gives slightly lower values of the drag coefficient up to a certain Reynolds number and then, the two curves coincide. For $M_\infty = 1$ and $v_0/U = -\cdot 001$, the exponential profile gives sufficiently higher values of the drag coefficient.

Comparing the results of the three profiles for the particular case $M_\infty = 0$ and $(v_0/U = 0)$ with the Blasius profile, we find that Lew's exponential profile overestimates the local shearing stresses by 19·2 per cent. and his quartic profile overestimates by 3·31 per cent. while the present quartic profile underestimates the local shearing stresses by 6·91 per cent.

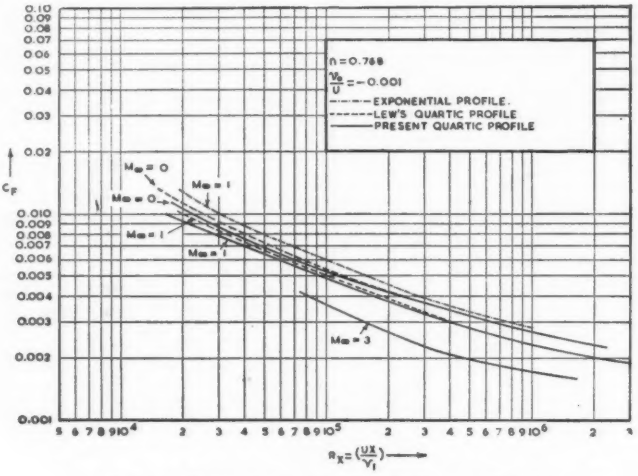


FIG. 6. Comparison of skin friction coefficients.

5. CONCLUSIONS

A comparative study of the three profiles have been made. The present quartic profile satisfies two boundary conditions at the plate and two at the top of the boundary layer, Lew's quartic profile satisfies one boundary condition at the plate and three at the outer edge of the boundary layer and the exponential profile satisfies one boundary condition at the plate and

infinite boundary conditions at $\tau = (t/\delta_t) = \infty$. The following conclusions have been derived:

(i) Lew's quartic profile hold in the limited range of Reynolds number while the results of our profile and exponential profile hold in the entire range of Reynolds number.

(ii) Characteristics of the flow, given by the exponential profile are considerably higher than the present quartic profile or Lew's quartic profile while these characteristics, given by our profile have lower values as compared to Lew's quartic profile up to a certain Reynolds number in the range of its applicability.

(iii) Nature of the characteristics of the flow is the same in all cases as we get in the case of impermeable wall, namely, with the increase of Mach number, the boundary layer thickness increases, the drag coefficient decreases and the velocity profiles become almost linear.

(iv) As the suction coefficient increases, boundary layer thickness decreases. This thinning of boundary layer results in delayed transition. Therefore, although the drag coefficient increases as the suction coefficient increases, the net drag on the body will be reduced due to delayed transition.

ACKNOWLEDGEMENT

The author is sincerely grateful to Prof. P. L. Bhatnagar, for his valuable advice and constant help throughout the preparation of this paper.

REFERENCES

1. Dorodnitzyn .. *Laminar Boundary Layer in Compressible Fluids*, Central Institute of Aerodynamics, Moscow, 1942.

2. Howarth, L. .. "On the calculation of steady flow in the boundary Layer near the surface of a cylinder in a stream," *R and M, British A.R.C. No.* 1632, 1934.

3. Kármán, Th. V. and Tsien, H. .. "Boundary layer in compressible Fluids," *J. of Aero. Sci.*, 1938, **5**, 227.

4. Kuerti, G. .. *Advances in Applied Mechanics*, 1951, **2**.

5. Lew, H. G. .. "On compressible boundary layer over a flat plate with uniform suction," Reissner Anniversary, 1949, **1**, 43–60. *Contributions to Applied Mech.*, J. W. Edwards, *Ann. Arbor*, Michigan, 43–60.

6. Pohlhausen, K. .. "Zur näherungs Weisen Integration der differential gleichung der laminaren Grenzschicht," *ZAMM.*, 1921, **1.**

7. Torda, T. Paul. .. "Boundary layer control by continuous surface suction or injection," *J. Math. and Physics*, 1952, **31**, 206–17.

KINETIC STUDIES IN ESTER HYDROLYSIS

Part X. The Alkaline Hydrolysis of Some Benzoic Esters in Mixed Solvents

By S. V. Anantakrishnan, F.A.Sc. and P. S. Radhakrishnamurti

(Contribution from the Department of Chemistry, Madras Christian College, Tambaram, S. India)

Received September 21, 1960

Introduction

In earlier parts of this series were reported the results of studies on a number of aliphatic esters and in particular of ethyl acetate in a variety of solvent compositions. Specific solvent influences were noticeable in which dielectric constant was only one of the factors and abnormal features were noticed where the organic solvent component in the aqueous system was a small fraction of the mixture.[1] It appeared necessary to study some aromatic esters as well as the esters of polybasic acids. The present report presents the results of studies with some benzoic esters.

Experimental

The solvents used were purified as in earlier parts while the esters were invariably freshly purified and distilled before use. The physical constants of the esters are given below:

Ethyl benzoate b.p. 212·9° C.	n_D^{30} 1·5028	Dipole moment	1·95
o-Nitro Ethyl benzoate b.p. 173·0/18 mm.	n_D^{30} 1·5211	,, ,,	3·79
p-Nitro Ethyl benzoate m.p. 57°		,, ,,	3·84
Benzyl benzoate b.p. 323°	n_D^{30} 1·5641	,, ,,	2·08

(All the physical constants were measured using the samples purified for the experiment. Dipole moment values refer to benzene solutions.)

The course of the reaction was followed by standard procedures, analysing aliquot parts. Freshly standardised dilute ammonia solutions were used for estimating excess acid used for chilling the reaction mixture and a screened indicator composed of neutral red and methylene blue was used. Temperature control was accurate to 0·02° C. in the range studied and was periodically checked.

Discussion of Results

Unlike the studies with ethyl acetate, the range of solvent compositions that could be used was limited by solubility of the reactants. In every instance, strict second order rate constants could be got and the graph of $\log (a - x)/(b - x)$ against time was linear within the limits of experimental error (Fig. 1). The relevant rate constant data are presented in Table I.

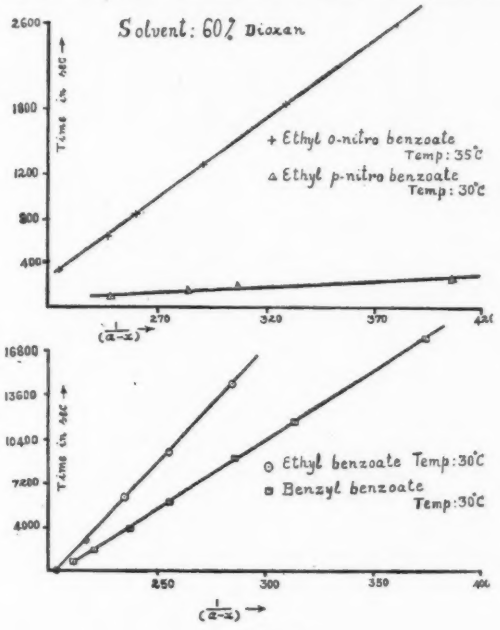


Fig. 1

It will be noticed that in all the instances, the reaction rate decreases with a decrease in dielectric constant, indicating repulsive interactions of the ion-dipole system if one is to assume what is not true, that free energies of hydration of the ion as well as of the transition state are unimportant. At the concentrations of reactants used, the Debye-Huckel limiting law is applicable and no serious error is introduced in applying the Kirkwood approximation.[4]

In dealing with mixed solvents, one has to choose between iso-composition and iso-dielectric conditions for comparisons. Svirbeley and Warner[1] object to the use of isocomposition data as such and use the relation.

$$E_D = E_{app} + R\left[\frac{\log K}{\partial D}\right]_T \frac{\partial D}{\partial\left(\frac{1}{T}\right)}.$$

TABLE I A

Ester	Solvent composition % dioxan v/v	Temperature °C. 30 Conc. of reactants × 10^3	30 k_2 × 10^3	35 Conc. of reactants × 10^3	35 k_2 × 10^3	40 Conc. of reactants × 10^3	40 k_2 × 10^3
Ethyl benzoate ..	40	5·005	9·72	8·560*	13·50	4·926	18·50
	50	4·908	7·42	8·532*	10·40	5·012	14·50
	60	5·026	5·95	8·521*	8·50	5·138	12·00
	70	5·192	5·08	8·516*	7·40	5·202	10·05
					$k_2 \times 10^2$		$lk_2 \times 10^2$
o-Nitro ethyl benzoate ..	40	..	..	5·017	10·87	5·025	14·41
	50	..	..	5·139	8·35	5·039	11·20
	60	..	..	4·957	6·84	5·024	9·26
	70	..	..	5·035	5·97	5·028	8·17
			$k_2 \times 10^2$				
p-Nitro ethyl benzoate ..	60	5·146	5·821	5·208	7·960	..	..
..	70	5·108	5·023	5·012	6·963	..	..
			$k_2 \times 10^3$		$k_2 \times 10^3$		$k_2 \times 10^3$
Benzyl benzoate ..	50	5·109	12·11	5·426	16·80	5·099	23·14
	60	5·047	9·92	5·229	14·00	5·008	19·50
	70	5·001	8·52	5·014	12·30	4·826	17·03

TABLE I B

The alkaline hydrolysis of benzyl benzoate in acetone-water system

Solvent composition Acetone% v/v	Temperature ° C.					
	30		35		40	
	Conc. of reactants $\times 10^3$ M	k_2 $\times 10^3$	Conc. of reactants $\times 10^3$ M	k_2 $\times 10^3$	Conc. of reactants $\times 10^3$	k_2 $\times 10^3$
50	4·824	9·31	4·926	13·40	5·209	19·04
60	5·226	7·95	5·014	11·57	5·129	16·50
70	5·098	5·901	5·024	8·70	4·999	12·69

Note.—The concentration of the hydroxide used was practically the same as that of the ester except those marked * where the OH′ concentration was about 0·001 M higher. All rate constants are in units of moles/litre, sec.$^{-1}$

The degree of linearity in the graph of the reciprocal of temperature against the logarithm of the rate constants in the present studies indicate that apparently the correction term is not appreciable and we are justified in using the more convenient isocomposition data in Table II.

Laidler and Eyring[5] have shown that in the alkaline hydrolysis of ethyl benzoate in alcohol-water systems, the empirical relationship reported by Evans and Polanyi[3] $T\Delta S = \alpha\Delta H + \beta$ holds good. While our results show clearly that the trends in both ΔH and ΔS appear to be similar, the expected linear relationship is not satisfactory. Apparently, where non-hydroxylic solvents are present, the solvation of the reactant species as well as the transition state are influenced to different degrees by the dielectric constant in the microscopic region as well as the topology of the transition state. The results, however, bring out the Fairclough-Hinshelwood relationship between the two Arrhenius parameters.[3] Even here, we have to treat each compound separately in a given solvent pair and changes in solvent pair systems also alter the individual values though the trend for a given compound may be maintained.

For an ion-dipole reaction, assuming a solvent continuum, it has been shown that the rates and activation energies involve functions of the dipole moments of the reacting species. The reaction centre in these esters is the COOR group and substituents in the benzene ring can modify the group

TABLE II

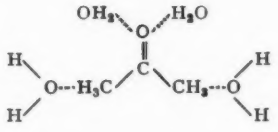
Ester	Solvent dioxan vol.%	E k.cals.	ΔH k.cals.	$\log_{11} PZ$	ΔS e.u.
Ethyl benzoate ..	40	12·220	11·608	6·800	−27·51
	50	12·760	12·148	7·300	−25·17
	60	13·230	12·618	7·600	−23·84
	70	13·950	13·338	7·760	−23·11
o-Nitro ethyl benzoate	40	10·800	10·818	6·696	−27·97
	50	11·260	10·648	6·908	−27·01
	60	11·610	10·998	7·070	−26·26
	70	12·010	11·398	7·293	−25·24
p-Nitro ethyl benzoate	60	11·570	10·948	8·102	−21·54
	70	12·280	11·668	8·552	−19·47
Benzyl benzoate ..	50	12·470	11·858	7·066	−26·27
	60	12·790	12·178	7·220	−25·59
	70	13·280	12·678	7·508	−24·25
	Acetone water system % Acetone by vol.				
	50	13·550	12·938	7·737	−23·21
	60	13·960	13·348	7·964	−22·17
	70	14·530	13·918	8·250	−20·91

moment by altering the apparent charge on the carbon of the carbonyl. The introduction of the nitro group can then be expected to reduce the activation evergy for the reaction as is actually found to be the case. The position in the benzene ring does not appear to make any serious difference to the activation energy but as may be expected, there is a pronounced difference in the entropy of activation. The value is considerably higher with

the ortho-nitro compound. The proximity of the negative end of the N–O dipole hinders the approach of the OH′ ion from certain directions. Alteration of the alkoxy group is not without effect and the replacement of the ethyl by benzyl leads to a slight reduction in the activation energy following the trend of the dipole moment.

Turning to solvent influence on the reaction, both rates and activation energies follow the trend in the dielectric constant with rates decreasing and activation energies increasing as the water content decreases.

It has been observed with ethyl acetate hydrolysis that abnormalities are noticed only for small additions of the organic solvent to water while in the region of compositions used in the present study, the characteristic correlation of Kirkwood[4] between Dielectric constant and rates holds good. The one compound studied in both dioxan-water and acetone-water system shows again the differences noticed in earlier parts. The reaction in acetone is consistently slower. The activation energy as well as the entropy of activation are also much higher and beyond the limits of experimental error. The difference has to be traced to the specific property of acetone in forming a solvated molecule of the type



so that the system as a whole has to use also the extra energy for rupture of the bond between water and acetone in addition to the normal energy needed for the reaction. The increase in value of the entropy of activation is also consistent with this structure breaking part of the mechanism.

Summary

The alkaline hydrolysis of some benzoic esters in dioxan-water mixtures as well as that of benzyl benzoate in acetone-water has been studied. The observations indicate a group influence consistent with the ease of formation of the transition state by interaction between the hydroxyl ion and the carbonyl group of the ester in the rate determining step. The influence of dielectric constant is what might be expected for an ion-dipole reaction but specific solvent influences cannot be excluded.

REFERENCES

1. Anantakrishnan and Anantaraman — *Proc. Ind. Acad. Sci.*, 1959, **49**, 86 and 174.
2. Evans and Polanyi .. *Trans. Farad. Soc.*, 1936, **38**, 133.
3. Fairclough and Hinshelwood *J.C.S.*, 1937, 538.
4. Kirkwood .. *J. Chem. Phys.*, 1934, **2**, 351.
5. Laidler and Eyring .. *Annals N.Y. Acad. Sci.*, 1940, **39**, 403.
6. Svirbeley and Warner .. *J.A.C.S.*, 1935, **57**, 1883.

SOME THEOREMS ON SELF-RECIPROCAL FUNCTIONS

BY S. MASOOD

(Department of Mathematics, Banaras Hindu University, Varanasi-5)

Received July 25, 1960

(Communicated by Dr. V. V. Narlikar, F.A.SC.)

IN a paper Brij Mohan[5] has given the following theorem connecting different classes of self-reciprocal functions:

If $f(x)$ is R_μ and

$$P(x) = \frac{1}{2\pi i}\int_{k-i\infty}^{k+i\infty} 2^s \Gamma(\tfrac{1}{4} + \tfrac{1}{2}\mu + \tfrac{1}{2}s)\,\Gamma(\tfrac{1}{4} + \tfrac{1}{2}\nu + \tfrac{1}{2}s)\,\omega(s)\,x^{-s}ds, \tag{1.1}$$

where

$$0 < k < 1, \quad \text{and} \quad \omega(s) = \omega(1-s), \tag{1.2}$$

then the function

$$g(x) = \int_0^\infty P(xy) f(y)\,dy \tag{1.3}$$

is R_ν.

Here we proceed to take up the converse problem, namely;

If $f(x)$ is R_μ and $g(x)$ is R_ν, what will be the character of the function $P(x)$ in (1.3)?

The solution of this problem is covered by the following:

THEOREM 1: In (1.3) if $f(x)$ is R_μ and $g(x)$ is R_ν, then $P(x)$ will be given by (1.1).

Multiplying (1.3) by x^{s-1} and integrating with respect to x between the limits 0 and ∞, we get

$$\int_0^\infty x^{s-1} g(x)\,dx = \int_0^\infty x^{s-1}dx \int_0^\infty P(xy) f(y)\,dy$$

$$= \int_0^\infty f(y)\,dy \int_0^\infty x^{s-1} P(xy)\,dx,$$

provided the change in the order of integration is justified. Therefore, we have

$$\int_0^\infty g(x)\, x^{s-1} dx = \int_0^\infty y^{-s} f(y)\, dy \int_0^\infty u^{s-1} \mathrm{P}(u)\, du. \tag{1.4}$$

Since $f(x)$ is R_μ, we have[3]

$$f(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} 2^{\frac{1}{2}s} \Gamma(\tfrac{1}{4} + \tfrac{1}{2}\mu + \tfrac{1}{2}s)\, \psi(s)\, x^{-s} ds,$$

where $0 < c < 1$ and $\psi(s)$ satisfies (1.2).

Hence

$$\int_0^\infty f(x)\, x^{s-1} dx = 2^{\frac{1}{2}s} \Gamma(\tfrac{1}{4} + \tfrac{1}{2}\mu + \tfrac{1}{2}s)\, \psi(s).$$

changing s into $1 - s$,

$$\int_0^\infty x^{-s} f(x)\, dx = 2^{\frac{1}{2}-\frac{1}{2}s} \Gamma(\tfrac{3}{4} + \tfrac{1}{2}\mu - \tfrac{1}{2}s)\, \psi(s),$$

since

$$\psi(s) = \psi(1 - s).$$

Similarly, as $g(x)$ is R_ν, we shall have[3]

$$\int_0^\infty x^{s-1} g(x)\, dx = 2^{\frac{1}{2}s} \Gamma(\tfrac{1}{4} + \tfrac{1}{2}\nu + \tfrac{1}{2}s)\, \chi(s),$$

where $\chi(s)$ satisfies (1.2).

Hence (1.4), with the help of the above known integrals, is easily reducible to the following integral

$$\int_0^\infty u^{s-1} \mathrm{P}(u)\, du = \frac{2^{\frac{1}{2}s} \Gamma(\tfrac{1}{4} + \tfrac{1}{2}\nu + \tfrac{1}{2}s)\, \chi(s)}{2^{\frac{1}{2}-\frac{1}{2}s} \Gamma(\tfrac{3}{4} + \tfrac{1}{2}\mu - \tfrac{1}{2}s)\, \psi(s)},$$

$$= 2^s \Gamma(\tfrac{1}{4} + \tfrac{1}{2}\mu + \tfrac{1}{2}s)\, \Gamma(\tfrac{1}{4} + \tfrac{1}{2}\nu + \tfrac{1}{2}s)\, \psi_1(s),$$

where

$$\psi_1(s) = \frac{\chi(s)}{2^{\frac{1}{2}} \Gamma(\tfrac{1}{4} + \tfrac{1}{2}\mu + \tfrac{1}{2}s)\, \Gamma(\tfrac{3}{4} + \tfrac{1}{2}\mu - \tfrac{1}{2}s)\, \psi(s)}$$

and hence satisfies the relation

$$\psi_1(s) = \psi_1(1-s).$$

Therefore, by Mellin's Inversion formula,[1] we have

$$\mathrm{P}(u) = \frac{1}{2\pi i}\int_{k-i\infty}^{k+i\infty} 2^s\Gamma(\tfrac{1}{4}+\tfrac{1}{2}\mu+\tfrac{1}{2}s)\,\Gamma(\tfrac{1}{4}+\tfrac{1}{2}\nu+\tfrac{1}{2}s)\,\psi_1(s)\,u^{-s}ds,$$

where $\psi_1(s)$ satisfies (1.2), which is the same as the function given by (1.1).

The generalisation of the above theorem, by the same author[7] is given by the following:

If $f(x)$ is of the form

$$f(x) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} 2^{2s}\Gamma(\tfrac{1}{4}+\tfrac{1}{2}\mu+\tfrac{1}{2}s)\,\Gamma(\tfrac{1}{4}+\tfrac{1}{2}\nu+\tfrac{1}{2}s)\,\Gamma(\tfrac{3}{4}+\tfrac{1}{2}\lambda-\tfrac{1}{2}s) \times \psi(s)\,x^{-s}ds, \qquad (1.5)$$

where $\psi(s)$ satisfies (1.2), and $\mathrm{P}(x)$ is given by (1.1), then the function $g(x)$ given by (1.3) belongs to R_λ.

The corresponding converse result will be:

THEOREM 1 (*a*): In (1.3) if $f(x)$ is given by (1.5) and $g(x)$ is R_λ, then $\mathrm{P}(x)$ will be given by (1.1).

This can be established in the same way as Theorem 1. Another problem of a similar nature is:

If $g(x)$ is R_ν and $\mathrm{P}(x)$ is of the form (1.1), what will be the character of the function $f(x)$?

THEOREM 2: In (1.3) if $g(x)$ is R_ν and $\mathrm{P}(x)$ is given by (1.1), then the function $f(x)$ will be of the form

$$f(x) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} 2^{2s}\Gamma(\tfrac{1}{4}+\tfrac{1}{2}\mu+\tfrac{1}{2}s)\,\psi(s)\,x^{-s}ds,$$

where $\psi(s)$ satisfies the relation (1.2), and hence will be R_μ.

The corresponding converse for the generalised theorem referred to above would be;

THEOREM 2 (*a*): In (1.3) if $g(x)$ is R_λ and $P(x)$ is of the form (1.1), then $f(x)$ will be given by (1.5).

The above theorems could be established in the same way as Theorem 1.

2. Another theorem due to the same author[5] states:

If $f(x)$ is R_μ and

$$P(x) = \frac{1}{2\pi i} \int_{k-i\infty}^{k+i\infty} \Gamma(\tfrac{1}{4} + \tfrac{1}{2}\mu + \tfrac{1}{2}s)\, \Gamma(\tfrac{3}{4} + \tfrac{1}{2}\nu - \tfrac{1}{2}s)\, \omega(s)\, x^{-s} ds, \quad (2.1)$$

where $0 < k < 1$, and $\omega(s)$ satisfies (1.2), then the function

$$g(x) = \int_0^\infty P(y) f(xy)\, dy \quad (2.2)$$

belongs to R_ν.

The converse theorem would be:

THEOREM 3: In (2.2) if $f(x)$ is R_μ and $g(x)$ is R_ν, then $P(x)$ will be given by (2.1).

In Theorem 3 if we put $\mu = \nu$, it will give rise to the following simple Corollary:

Corollary.—In (2.2) if $f(x)$ is R_ν, and $g(x)$ is R_ν, then $P(x)$ will be given by

$$P(x) = \frac{1}{2\pi i} \int_{k-i\infty}^{k+i\infty} \lambda(s)\, x^{-s} ds,$$

where $\lambda(s)$ satisfies (1.2).

In another paper Brij Mohan[7] generalised the above theorem in the following form:

If $f(x)$ is of the form

$$f(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} 2^{\frac{1}{2}s} \Gamma(\tfrac{1}{4} + \tfrac{1}{2}\mu + \tfrac{1}{2}s)\, \Gamma(\tfrac{3}{4} + \tfrac{1}{2}\nu - \tfrac{1}{2}s) \times \Gamma(\tfrac{1}{4} + \tfrac{1}{2}\lambda + \tfrac{1}{2}s)\, \chi(s)\, x^{-s} ds, \quad (2.3)$$

where $\chi(s)$ satisfies (1.2), then the function given by (2.2) is R_λ provided that $P(x)$ is given by (2.1).

The corresponding converse theorem will be:

THEOREM 3 (*a*): In (2.2) if $f(x)$ is of the form (2.3) and $g(x)$ is $\mathbf{R}_\lambda$, then $\mathrm{P}(x)$ is given by (2.1).

When $\mu = \nu$, in (2.3), the corollary given under Theorem 3 also holds.

We have two more similar theorems:

THEOREM 4: In (2.2) if $g(x)$ is $\mathbf{R}_\nu$ and $\mathrm{P}(x)$ is of the form (2.1), then $f(x)$ will be given by

$$f(x) = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} 2^{is}\Gamma(\tfrac{1}{4} + \tfrac{1}{2}\mu + \tfrac{1}{2}s)\,\psi(s)\,x^s\,ds,$$

where $\psi(s) = \psi(1-s)$, and hence will be $\mathbf{R}_\mu$.

If $\mu = \nu$, in Theorem 4, we get the following Corollary:

Corollary.—In (2.2) if $g(x)$ is $\mathbf{R}_\nu$ and

$$\mathrm{P}(x) = \frac{1}{2\pi i} \int_{k-i\infty}^{k+i\infty} \lambda(s)\,x^{-s}ds,$$

where $\lambda(s) = \lambda(1-s)$, then the function $f(x)$ will also be $\mathbf{R}_\nu$.

THEOREM 4 (*a*): In (2.2) if $g(x)$ is $\mathbf{R}_\lambda$ and $\mathrm{P}(x)$ is of the form (2.1), then $f(x)$ will be given by (2.3).

When $\mu = \nu$, a corollary similar to that given above will be obtained.

The proofs of Theorems 3, 3 (*a*), 4 and 4 (*a*) are similar to that of the theorem established above.

3. In another paper the same author[6] has given the following theorem.

If $f(x)$ is $\mathbf{R}_\mu$, the function

$$g(x) = \frac{1}{x} \int_0^\infty \mathrm{Q}\left(\log \frac{x}{y}\right) f(y)\,dy \qquad (3\cdot1)$$

is R_ν, provided that

$$Q(x) = \frac{1}{2\pi i}\int_{k-i\infty}^{k+i\infty} \Gamma(\tfrac{1}{4}+\tfrac{1}{2}\mu+\tfrac{1}{2}s)\,\Gamma(\tfrac{3}{4}+\tfrac{1}{2}\nu-\tfrac{1}{2}s)\,\omega(s)\,e^{xs}ds \quad (x>0)$$

$$= 0 \qquad (x<0), \tag{3.2}$$

where $k > 0$ and $\omega(s)$ satisfies

$$\omega(s) = \omega(1-s). \tag{3·3}$$

The converse problem here would be:

If $f(x)$ is R_μ and $g(x)$ is R_ν, what will be the character of the function $Q(x)$ in (3.1)?

The solution of this problem is covered by the following:

THEOREM 5: In (3.1) if $f(x)$ is R_μ and $g(x)$ is R_ν, then $Q(x)$ will be given by (3.2).

Substituting u for $\log(x/y)$ in (3.1), we get

$$g(x) = \int_0^\infty Q(u)\,f(xe^{-u})\,e^{-u}du.$$

Multiplying this equation by x^{s-1}, and integrating with respect to x, between the limits 0 and ∞, we have:

$$\int_0^\infty g(x)\,x^{s-1}dx = \int_0^\infty x^{s-1}dx \int_0^\infty Q(u)\,f(xe^{-u})\,e^{-u}du.$$

Let $x = ye^u$.

Substituting this value for x on the R.H.S. and rearranging the terms we get,

$$\int_0^\infty x^{s-1}g(x)\,dx = \int_0^\infty f(y)\,y^{s-1}dy \int_0^\infty Q(u)\,e^{(s-1)u}du, \tag{3.4}$$

provided the change in the order of integration is justified. Given that the functions $f(x)$ and $g(x)$ are R_μ and R_ν respectively, we have[3]

$$\int_0^\infty x^{s-1}f(x)\,dx = 2^{1s}\Gamma(\tfrac{1}{4}+\tfrac{1}{2}\mu+\tfrac{1}{2}s)\,\psi(s),$$

and

$$\int_0^\infty x^{s-1} g(x)\, dx = 2^{\frac{1}{2}s}\Gamma(\tfrac{1}{4} + \tfrac{1}{2}\nu + \tfrac{1}{2}s)\,\chi(s),$$

where both $\psi(s)$ and $\chi(s)$ satisfy the relation (1.2).

Substituting these values for the respective integrals in (3.4), we get

$$\int_0^\infty Q(u)\, e^{(s-1)u} du = \frac{\Gamma(\tfrac{1}{4} + \tfrac{1}{2}\nu + \tfrac{1}{2}s)\,\chi(s)}{\Gamma(\tfrac{1}{4} + \tfrac{1}{2}\mu + \tfrac{1}{2}s)\,\psi(s)}.$$

changing s into $(1 - s)$,

$$\int_0^\infty Q(u)\, e^{-su} du = \frac{\Gamma(\tfrac{3}{4} + \tfrac{1}{2}\nu - \tfrac{1}{2}s)\,\chi(s)}{\Gamma(\tfrac{3}{4} + \tfrac{1}{2}\mu - \tfrac{1}{2}s)\,\psi(s)},$$

$$= \Gamma(\tfrac{1}{4} + \tfrac{1}{2}\mu + \tfrac{1}{2}s)\,\Gamma(\tfrac{3}{4} + \tfrac{1}{2}\nu - \tfrac{1}{2}s)\,\psi_1(s),$$

where

$$\psi_1(s) = \frac{\chi(s)}{\Gamma(\tfrac{1}{4} + \tfrac{1}{2}\mu + \tfrac{1}{2}s)\,\Gamma(\tfrac{3}{4} + \tfrac{1}{2}\mu - \tfrac{1}{2}s)\,\psi(s)},$$

and hence satisfies the relation $\psi_1(s) = \psi_1(1 - s)$.

Hence, by applying another type of Mellin's Inversion Formula,[2] we obtain

$$Q(x) = \frac{1}{2\pi i}\int_{k-i\infty}^{k+i\infty} \Gamma(\tfrac{1}{4} + \tfrac{1}{2}\mu + \tfrac{1}{2}s)\,\Gamma(\tfrac{3}{4} + \tfrac{1}{2}\nu - \tfrac{1}{2}s)\,\psi_1(s)\, e^{xs} ds \quad (x > 0)$$

$$= 0 \qquad (x < 0),$$

which is the same as (3.2).

In another paper[4] the present writer has generalised the theorem due to Brij Mohan in the following form:

If $f(x)$ is of the form

$$f(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} 2^{\frac{1}{2}s}\Gamma(\tfrac{1}{4} + \tfrac{1}{2}\mu + \tfrac{1}{2}s)\,\Gamma(\tfrac{3}{4} + \tfrac{1}{2}\nu - \tfrac{1}{2}s)$$

$$\times \Gamma(\tfrac{1}{4} + \tfrac{1}{2}\lambda + \tfrac{1}{2}s)\,\psi(s)\, x^{-s} ds, \qquad (3.5)$$

where $0 < c < 1$ and $\psi(s)$ satisfies the relation (1.2), the function $g(x)$ given by (3.1) belongs to R_λ, provided that (3.2) holds.

The corresponding converse result for this theorem would be:

THEOREM 5 (*a*): In (3.1), if $f(x)$ is of the form (3.5), and $g(x)$ is R_λ, then $Q(x)$ will be given by (3.2).

The proof of this theorem is similar to that of the above theorem.

Another problem of a similar nature is covered by the following.

THEOREM 6: In (3.1), if $g(x)$ is R_ν and $Q(x)$ is given by (3.2), then $f(x)$ is R_μ,

and the corresponding result for the above generalised theorem would be:

THEOREM 6 (*a*): In (3.1), if $g(x)$ is R_λ and $Q(x)$ is of the form (3.2) then $f(x)$ will be given by (3.5).

The proofs of these two theorems run parallel to that of Theorem 5 already proved.

4. Another theorem due to Brij Mohan[6] is as follows:

If $f(x)$ is R_μ, the function

$$g(x) = \int_0^{1/a} Q\left(\log \frac{1}{xy}\right) f(y)\, dy \tag{4.1}$$

is R_ν provided that

$$Q(x) = \frac{1}{2\pi i} \int_{k-i\infty}^{k+i\infty} 2^s \Gamma\left(\tfrac{1}{4}+\tfrac{1}{2}\mu+\tfrac{1}{2}s\right) \Gamma\left(\tfrac{1}{4}+\tfrac{1}{2}\nu+\tfrac{1}{2}s\right) \omega(s)\, e^{xs} ds \quad (x > 0)$$

$$= 0 \qquad (x < 0), \tag{4.2}$$

where $k > 0$ and $\omega(s)$ satisfies the relation (1.2).

The converse problem is covered by the following

THEOREM 7: In (4.1), if $f(x)$ is R_μ (R_ν) and $g(x)$ is R_ν (R_μ) then $Q(x)$ is given by (4.2).

The present writer[4] has generalised the above theorem of Brij Mohan in the following form:

If $f(x)$ is a function of the form

$$f(x) = \frac{1}{2\pi i}\int_{b-i\infty}^{b+i\infty} 2^{1s}\Gamma(\tfrac{1}{4}+\tfrac{1}{2}\mu+\tfrac{1}{2}s)\ \Gamma(\tfrac{1}{4}+\tfrac{1}{2}\nu+\tfrac{1}{2}s)$$
$$\times\ \Gamma(\tfrac{1}{4}+\tfrac{1}{2}\lambda-\tfrac{1}{2}s)\,\psi(s)\,x^{-s}ds, \qquad (4.3)$$

where $0 < b < 1$, and $\psi(s) = \psi(1-s)$,

then the function given by (4.1) is R_λ, provided that $Q(x)$ is given by (4.2).

The corresponding converse result to this theorem would be:

THEOREM 7 (*a*): In (4.1), if $f(x)$ is of the form (4.3), and $g(x)$ is R_λ, then $Q(x)$ is given by (4.2).

Two more converse results of a similar nature corresponding to Theorems 6 and 6 (*a*) for the above-mentioned theorems of Brij Mohan and the present writer would be:

THEOREM 8: In (4.1), if $g(x)$ is R_ν and $Q(x)$ is given by (4.2), then $f(x)$ is R_μ.

THEOREM 8 (*a*): In (4.1) is $g(x)$ is R_λ and $Q(x)$ is of the form (4.2) then $f(x)$ will be given by (4.3).

The theorems of this article can be proved in the same way as those of article 3.

I offer my thanks to Dr. Brij Mohan for his help and guidance in the preparation of this note.

REFERENCES

1. Hardy, G. H. .. "On Mellin's inversion formula," *Mess. of Maths.*, 1918, **47**, 178.
2. ———— .. "Further notes on Mellin's inversion formulæ," *Ibid.*, 1921, **50**, 166.
3. ——— and Tithmarsh, E. C. "Self-reciprocal functions," *Quart. J. of Maths.* (*Oxf. series*), 1930, **1**, 208.
4. Masood, S. .. "Some formulæ connecting self-reciprocal functions," (In the Press).
5. Mohan, B. .. "Some theorems on self-reciprocal functions," *Proc. Lond. Math. Soc.*, 1932, **34**, 231–40.
6. ———— .. "Theorems connecting different classes of self-reciprocal functions," *Proc. Edin. Math. Soc.*, 1934, **4** (2), 53–56.
7. ———— .. "On self-reciprocal functions," *Quart. J. of Maths.* (*Oxf, series*), 1939, **10**, 252–60.

ON THE LOGICAL AND CIRCUIT DESIGN OF THE DIGITAL COMPUTER AT T.I.F.R., BOMBAY

By B. K. Basu* and S. P. Srivastava

(*Tata Institute of Fundamental Research, Bombay-1*)

Received July 27, 1960

(Communicated by Dr. S. S. Dharmatti, F.A.SC.)

Introduction

The T.I.F.R. computer is a parallel, binary, asynchronous machine. It is controlled by a stored programme of single address instructions and has a fast access ferrite core memory, consisting of 1024 locations. The input to the computer is by means of a punched paper tape (5-hole commercial teletype tape) and the output can be either printed out directly or punched on a paper tape again.

In an earlier paper,[1] the principal features of the engineering and system design of this computer were dealt with. In this paper we shall consider in some detail the logical and circuit design of its arithmetic unit, control and memory. While some knowledge of the logical design and organisation of digital computer is assumed on the part of the reader, this paper is self-contained as regards the design of the T.I.F.R. machine.

The Arithmetic Logic and the Circuitry

In the T.I.F.R. computer, all the internal operations take place in the binary notation and numbers are stored in the main memory as signed binary fractions restricted to the range $(-1, 1)$. A number in its normal form consists of a sign (which is 0 for + ve numbers and a 1 for −ve numbers) followed by 39 binary digits (bits for short) which give its absolute value as a binary fraction. The binary point is understood to be fixed immediately after the sign bit. The complement form of a number is obtained by changing all its 1's to 0's and *vice versa*, the sign bit is left unaltered in the process.

A schematic diagram of the arithmetic unit is shown in Fig. 1. It consists of a memory register, two (double rank) shift registers (the Accumulator and M.Q. register respectively), an adder and a pair of true/compl. gates. The contents of the accumulator are presented to the adder either in the true or the complement form, depending on whether the operation called for is addi-

* Now on study leave at Brown University, U.S.A.

tion (Multiplication) or subtraction (Division), respectively. In some cases the result of a subtraction needs to be recomplemented. To achieve this, facilities have been built into the adder so that its output can be gated into the accumulator either in its true or complement form.

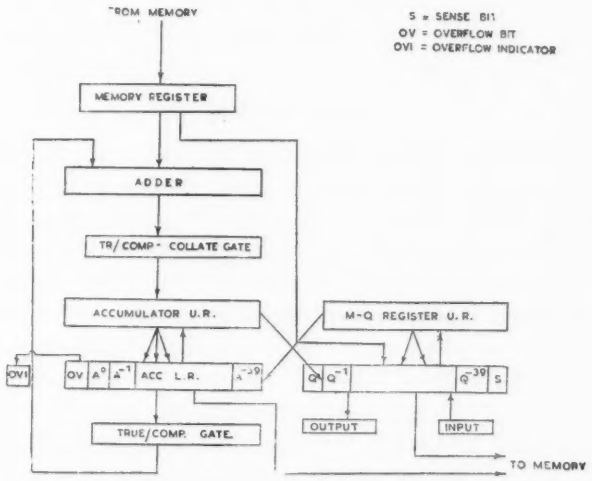


FIG. 1. Schematic Diagram of Arithmetic Unit Asymmetrical.

The three registers, Memory register, Accumulator and M.Q., are essentially storage units for holding the numbers involved in the arithmetic operations. The basic flip-flop used is of the asymmetrical type as shown in Fig. 2. By definition the flip-flop represents the binary digit 1 when the left hand triode is cut off and right hand one is conducting. As seen from Fig. 2 the output voltage at P for '1' and '0' is + 30 and −20 volts respectively. The transfer of information is achieved by first clearing, *i.e.*, making the flip-flop '0' and then gating. Clearing is done by dropping the plate supply (+ 110 Dr) from + 110 V to about + 40 V. The T.I.F.R. machine being parallel, it is necessary for all the 40 flip-flops of a register to be cleared simultaneously. This is done by giving the + 110 supply through a driver whose output goes down to + 40 V when pulsed. Gating is done by applying a + ve pulse rising from −100 V to −10 V. Depending on whether the input is '1' (+ 30 V) or '0' (−20 V) the flip-flop changes to the state '1' or '0' respectively. The clear and gate drivers are discussed in detail later on.

As shown in Fig. 1, the accumulator is a double rank (41 bits) register. The extra bit is the overflow bit (*cf.* below). It consists of an Upper Register

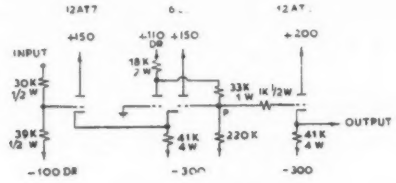


FIG. 2. Flip-flop.

and a Lower Register. Provision is made for shift UP (the contents of the Lower Register to the Upper Register), shift Dn (the contents of the Upper Register to the lower Register), shift left (the contents of $2^{-(n+1)}$ Upper Register to 2^{-n} Lower Register) and shift right (the contents of $2^{-(n-1)}$ Upper Register to 2^{-n} Lower Register) and gate adder to the Upper Register.

M.Q. is also a double rank 41 digit Register capable of shift UP, shift left, shift right and gate from Memory Register to Lower Register. The extra bit is placed as the 2^{-40} bit. It serves as the sense bit.

The adder is of the fast logical type and incorporates in itself special carry by-pass facilities[2] to speed up the carry propagation time. In an adder the factor responsible for serial propagation of carry is given by the logical equation

$$C_n = C_{n-1} \cdot (A_n + B_n)$$

where A_n and B_n are the inputs (corresponding to 2^{-n} bit) to the adder from Memory Register and Accumulator and C_{n-1} is the carry from the previous stage. The above expression means that C_n can be formed from C_{n-1} if either of the digit inputs is present. Rewriting the above equation

$$\begin{aligned} C_n &= C_{n-1}\ (A_n + B_n) \\ &= C_{n-2}\ (A_{n-1} + B_{n-1})\ (A_n + B_n) \\ &\quad \ldots\ldots\ldots\ldots \\ &\quad C_p\ (A_{p+1} + B_{p+1})\ \ldots\ldots\ldots (A_n + B_n). \end{aligned}$$

The carry output C_p from a lower digit stage P, can cause C_n in case at least one dilgit input in all the intermediate stages is present. If means are provided for C_p to sense in parallel the digit inputs of all the intermediate stages $(p+1)$, $(p+2)$, ... n, C_p will cause C_n without any delay. One can achieve this for n varying from 2 to N and p from 1 to $n-1$ (for an N stage adder), in which case no carry delay should occur. This, however,

is very cumbersome for N = 40 (the word length in the computer), considering the amount of electronic circuitry involved. But a compromise can be reached as in Fig. 3 which shows the carry by-pass logic and Fig. 4 which gives the circuit diagram as incorporated in the T.I.F.R. computer.

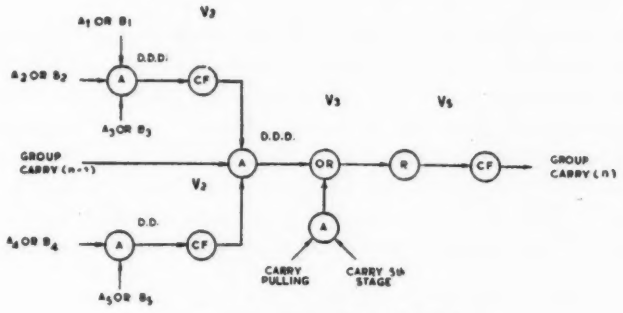


FIG. 3. Carry By-pass Logic.

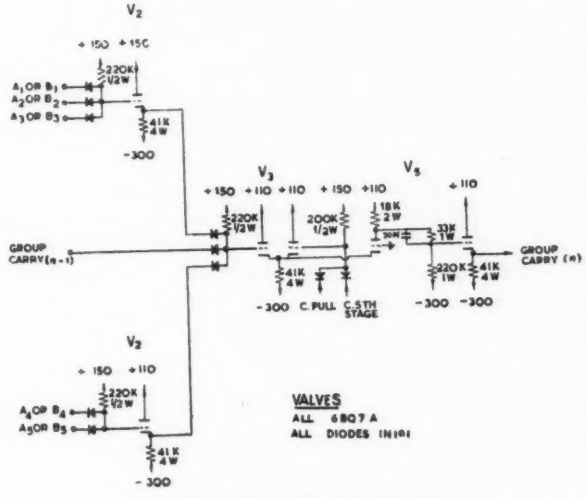


FIG. 4. Carry By-pass Circuit.

A number of 5 adder units are grouped together. There are 8 groups, *i.e.*, 8 carry by-pass units, the last group having only 4 units. Within this group carry propagation is serial, whereas a carry input incident on the least significant stage of the group can pass on to the next group directly in case at least one of the two digit inputs of every stage in the group is present. The

maximum carry delay occurs when carry is generated in the first stage of the first group, and dies down in the last stage of the last group, after passing through 4 stages, by-passing 6 groups and again going through 3 more stages. Assuming that one by-passing introduces the same delay as that of a single digit stage, the maximum carry delay is 13 times the carry delay per stage. In the present adder the maximum carry delay is 3μ sec.

All proper arithmetic operations should result in numbers which again fall within the range $(-1, 1)$. But there are no built-in checks to detect improper arithmetic operations. An extra bit, called the overflow bit, has been provided at the left end of the accumulator. In the case of genuine overflow (*i.e.*, addition of numbers with the same sign, or subtraction of numbers with opposite sign), it is fed into the overflow bit and an overflow indicator is set to 1. A "jump on overflow" order enables the programmer to make use of this information in controlling the details of his computations.

THE CONTROL LOGIC AND THE SEQUENCING UNITS

As has already been mentioned, the T.I.F.R. computer is of the asynchronous, single address type. The details of its control logic are effectively determined by these two features. A coded instruction in the machine requires 20 bits for its specification (the first 9 bits for its order part and last 11 bits for its address part). Because the word length in the memory is 40 bits, instructions are stored in pairs in each memory location. They are identified as the left hand (L.H.I.) and right hand (R.H.I.) instructions in that location. In magnetic drum orders two addresses are necessary—one for the internal memory and the other for the drum. Hence a complete word will be used for a drum order. A complete control cycle consists of 3 parts: (1) an *instruction cycle*, where a (next) pair of instructions is brought from the memory to the instruction register, (2) an *execution cycle A*, where the left hand instruction is executed and (3) an *execution cycle B*, where the right hand instruction is executed.

Since the number of orders is much less than 2^9, the complications of complete decoding and recoding are avoided and static control is obtained by partial decoding of the order part.

There are four sequencing units similar to the one used in ORDVAC type machines. In each sequencing unit, 2 flip-flops control the micro operations. Four different combinations of these 2 flip-flops are possible, *viz.*, 11, 01, 00, 10 and the micro operations referring to these combinations for a particular sequencing unit, the Add-Shift unit (Fig. 5) will be described here in detail as a typical example of control logic design. Table I shows the

logic and polarity of the signals coming from the static control to this Add-Shift sequencing unit.

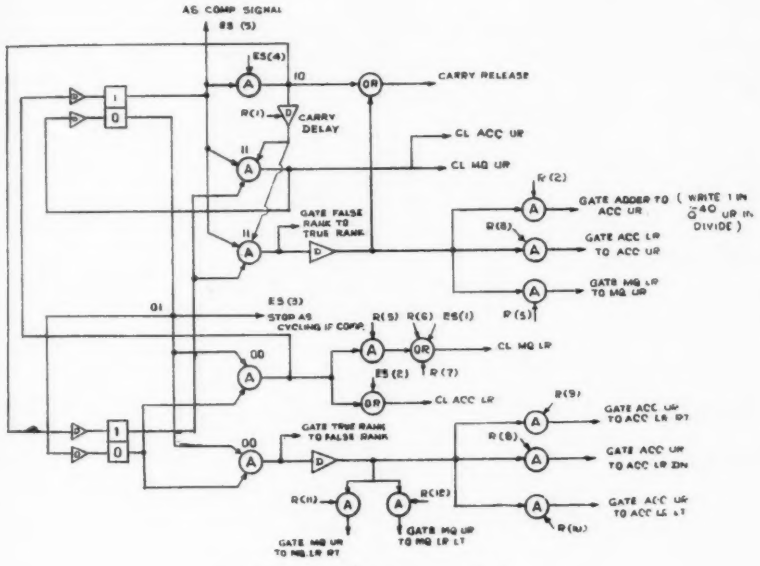


FIG. 5. **Add-Shift Sequence A–S.**

A–S sequence starts with 10 when it receives a signal from the previous sequencing unit, (E.S. 4) it signals the Carry Release Driver. Here it may be mentioned that in order to have fast dying down of the carry level and to avoid persistence of the end-around-carry level in the case of division, carry to all the stages of the adder is pulled down by means of a driver and is released as A–S moves and is again pulled down when gating from Adder to accumulator Upper Register is complete. The same signal changes the state of the sequencing unit to 11 after a delay as shown in the diagram. Depending on R_1, carry delay is initiated and 11 gives signal only when this delay is over. This signal goes to Clear Accumulator Upper Register Driver and clear M.Q. U.R. driver and after a delay changes the state of the sequencing unit to 01. This delay determines the pulse width of the clear pulse which in this computer has been made approximately 2μ sec.

The gate pulse, depending on R_2, R_3 and R_5, also starts at the same time as the clear pulse but is made to stay on for nearly 1μ sec. longer than the latter due to the delay shown after the 11 'And' gate.

TABLE I

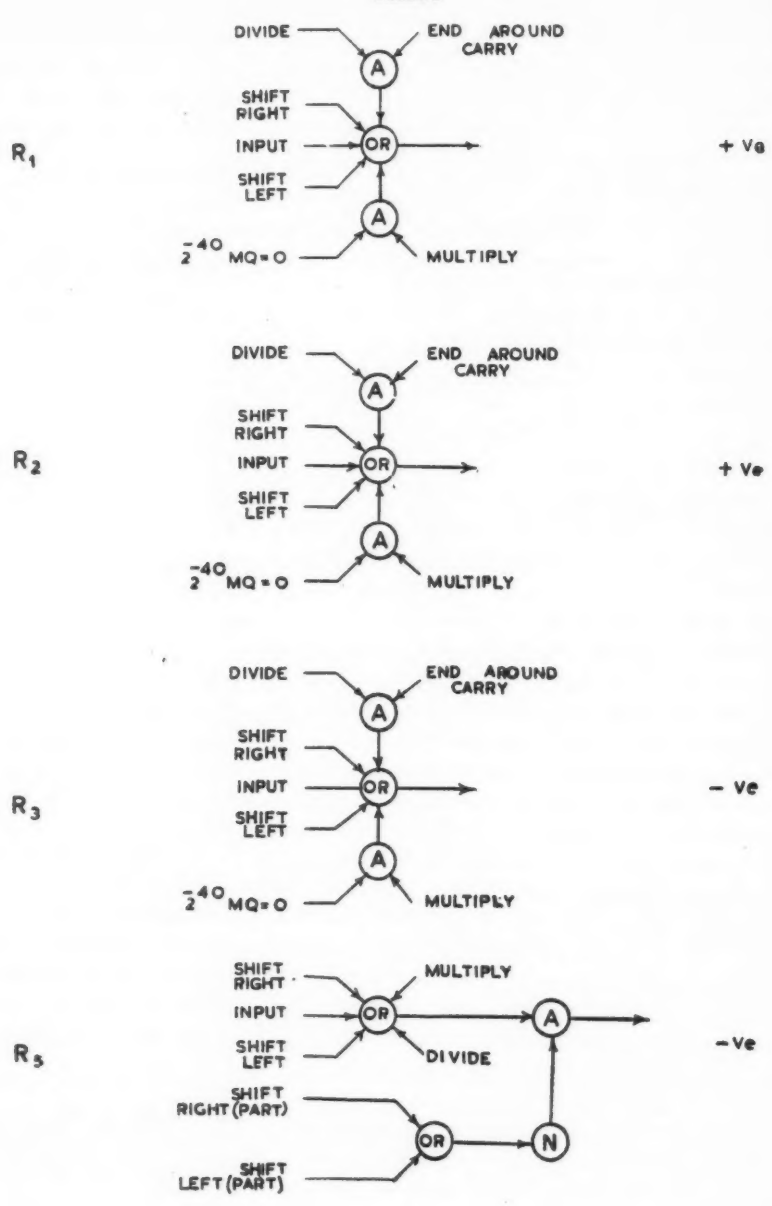
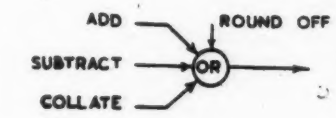
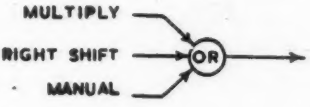
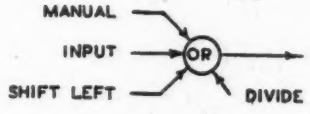
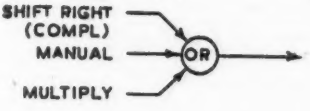
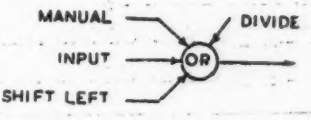

DIVIDE
END AROUND CARRY
A
SHIFT RIGHT
INPUT
OR
SHIFT LEFT
A
2^{-40} MQ = 0
MULTIPLY
R_1
+ Ve
DIVIDE
END AROUND CARRY
A
SHIFT RIGHT
INPUT
OR
SHIFT LEFT
A
2^{-40} MQ = 0
MULTIPLY
R_2
+ Ve
DIVIDE
END AROUND CARRY
A
SHIFT RIGHT
INPUT
OR
SHIFT LEFT
A
2^{-40} MQ = 0
MULTIPLY
R_3
- Ve
SHIFT RIGHT
MULTIPLY
INPUT
OR
A
SHIFT LEFT
DIVIDE
SHIFT RIGHT (PART)
OR
N
SHIFT LEFT (PART)
R_5
-Ve

TABLE I—(*Contd.*)

R_6		– Ve
R_8		– Ve
R_9		– Ve
R_{10}		– Ve
R_{11}		– Ve
R_{12}		– Ve

The 11 ' And ' gate feeds the pulse to the shift counter to keep track of the number of shifts. This shift counter[3] is a double rank counter similar to the one used in ORDVAC but incorporating an improved gating logic. A " recognition " circuit associated with the shift counter, gives the " recognition " signal to signify the end of the Add-Shift operation. Thus in the case of shift orders, the recognition is obtained as soon as the number of shifts, as specified in the address part of the instruction, has been accomplished and after 40 shifts in the case of multiplication and division orders. In the case of Add, Subtract and Collate orders the A–S sequence is allowed to go through its cycle only once.

As in the case of 11, the clear Acc. L.R. and M.Q. L.R. and the corresponding gate pulses are obtained from the state 00 of this sequencing unit. The complete micro operations of the A–S sequencing unit can be listed as follows:—

The state of A–S Unit	Micro operations performed
1 0/1 ..	(*a*) release the carry pulling.
	(*b*) delay the change of A–S to 11 if R_1 present (if R_1 –ve).
1 1 ..	(*a*) clear Acc. U.R.
	(*b*) clear M.Q. U.R.
	(*c*) if R_2, gate adder output to Acc. U.R. In case of divide and no end around carry, write 1 in 2^{-40} (M.Q.) U.R.
	(*d*) if R_3, gate Acc. L.R. to Acc. U.R.
	(*e*) if R_5, gate M.Q. L.R. to M.Q. U.R.
	(*f*) gate shift counter from False rank to True rank.
0 1/0 ..	(*a*) apply the carry pulling.
	(*b*) goes to the other sequencing unit (Execution Sequence) where it is used to stop A–S cycling depending on recognition.
0 0 ..	(*a*) Cl Acc. L.R.
	(*b*) Cl. M.Q. L.R. if R_5.
	(*c*) if R_8, gate Acc. U.R. to Acc. L.R. (Down).
	(*d*) if R_9, gate Acc. U.R. to Acc. L.R. (Right).

The state of A–S Unit	Micro operations performed
	(*e*) if R_{10}, gate Acc. U.R. to Acc. L.R. (Left).
	(*f*) if R_{11}, gate M.Q. U.R. to M.Q. L.R. (Right).
	(*g*) if R_{12}, gate M.Q. U.R. to M.Q. L.R. (Left).
	(*h*) gate the shift counter from true rank to false rank. Taking the case of the multiply order with 2^{-40} bit equal to 0 (*i.e.*, the sense bit = 0).
A–S 1 0/1	.. releases the carry pulling. It does not wait for carry delay.
A–S 1 1	.. clears Acc. U.R., clears M.Q. U.R.
	As R_8 is present from the logic in Table I, Acc. L.R. is gated to Acc. U.R. and as R_5 is present M.Q. L.R. is gated to M.Q. U.R.
A–S 0 0	.. clears Acc. L.R., clears M.Q. L.R. because R_5 is present. As R_9 and R_{11} are present, Acc. U.R. and M.Q. U.R. are gated "Right-Down" to Acc. L.R. and M.Q. L.R. respectively.
	So the whole A–S cycle shifts the contents of Acc. and M.Q. Lower Register one place to the right.

Taking the case of multiply with $2^{-40} = 1$ (*i.e.*, sence bit = 1), it can be seen that as R_1 is present, the carry delay will be initiated and as R_2 is present in place of R_3, after the carry delay, A–S 11 will gate the contents of the adder to Acc. U.R. Other micro operations will be same as in multiply with $2^{-40} = 0$.

So the whole A–S cycle adds the contents of Memory Register to Acc. and then shifts the contents of Acc. and M.Q. Lower Register one place to the right.

The other sequencing units perform their micro operations in an exactly similar fashion. In addition to the Add-Shift unit described above, there are 4 other sequencing units, one for the instruction cycle, one for the execution cycle, one for the output control and the last one called the Main Control, to control all the sequencing units.

CLEAR AND GATE DRIVERS

As mentioned earlier, clearing of the flip-flops (Fig. 2) is done by lowering the plate voltage of the left hand tube, thus lowering the grid of the right hand tube to cause it to cut off, and the left hand tube to become conducting. The gating is done by raising the common cathode of the flip-flop to cause the left tube to cut off. When the cathode is again lowered, the volage of the right grid is higher than that of the left grid. The right tube now is conducting and the left one is cut off. Clear and gate driver circuits[4] are similar to those described by J. C. Chu of Argonne National Laboratory for the Oak Ridge Automatic Computer. In order to minimise fluctuations of the driver output voltages, self-regulating drivers are used.

For True/Complement gate and for carry pulling, drivers (+ 40 V)/(−30 V) are used. They are also of the self-regulating type. Depending on the input, the output is constant either at + 40 V or − 30 V. Figure 6 shows the circuit diagram of this driver.

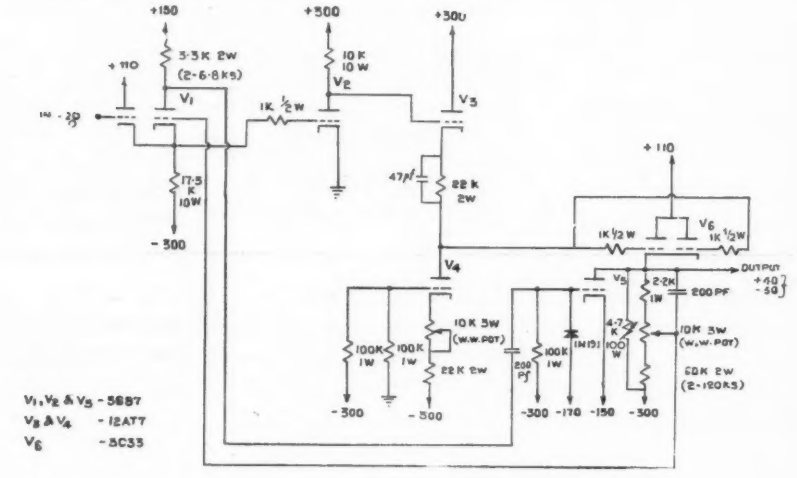


FIG. 6. + 40/− 30 Driver.

THE MEMORY

The internal memory of the T.I.F.R. computer consists of a 3-dimensional ferrite core matrix of 1024 words each of 40 digits. Figure 7 shows the schematic diagram of the memory.

As seen from the diagram, the memory consists of—apart from the store — X and Y diode decoding matrix for address selection, 40 output ampli-

fiers with amplitude and time discrimination for gating the output of memory cores into the Memory Register of the Arithmetic Unit, and read and write

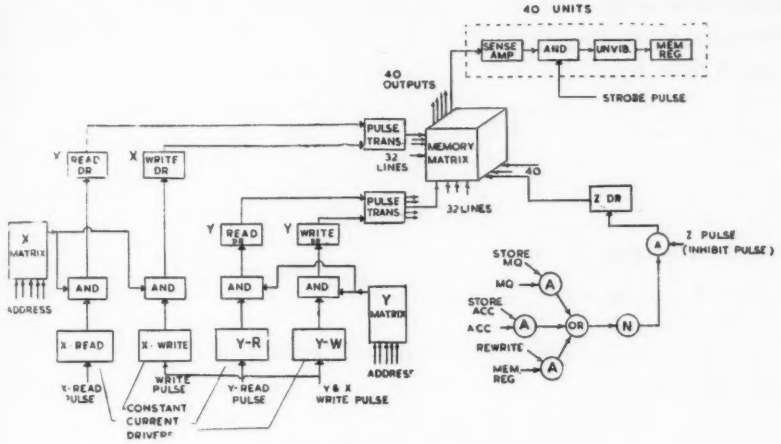


FIG. 7. Schematic Diagram of Memory.

circuits. The input to the memory can be either from the accumulator or the M.Q. or the Memory Register. In the case of the Acc. either the contents of the complete register or only the address part (either the left hand or the right hand address alone or both) can be written in. Since the memory read out is always destructive, in case it is required to rewrite the contents back, this is done from the Memory Register after the contents have been gated into it.

Read and write logic is essentially the same as that described by Papian for the M.I.T. computer.[5] Figure 8 shows the X-read, Y-read, Strobe, Write and Z (Inhibition) pulses. As can be seen the Y-read pulse starts after a slight delay from the X-read and the strobing is done after a further delay. The timings are shown in the figure.

Figure 9 shows one of the four constant current read and write Drivers along with its pulse transformer. The diode matrix decoder supplies + 100 V at the selected terminal and + 60 V at the unselected terminals. These potentials are fed to the grids of the driver valves having pulse transformer in the plate circuit. All the driver cathodes through small potentiometers are joined together and kept at + 130 V by a feed back stabilised system. All the driver valves are therefore normally cut off and no current flows through the pulse transformer. A current, slightly more than the driving current required, flows through the constant current tube. The read or write

pulse of about 30 V at the input, pulls down the grid of the constant current tube to 60 V. The common cathode is, however, arrested by the selected

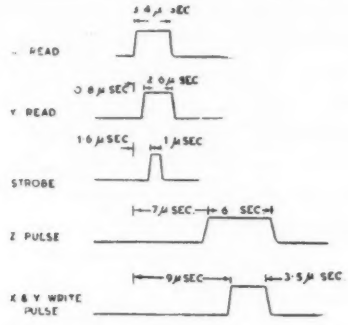


FIG. 8. Pulse Sequence in Memory.

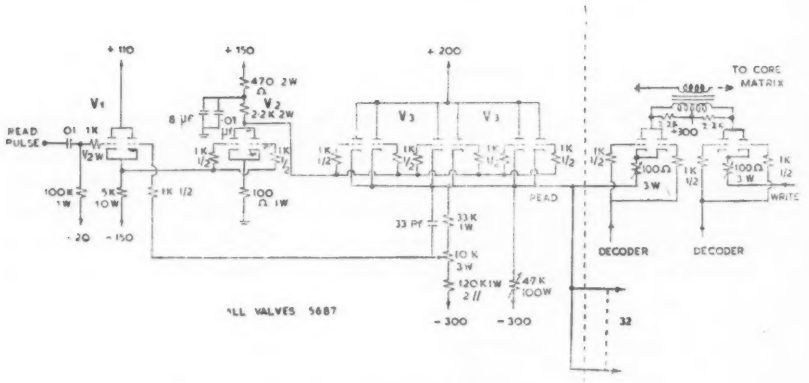
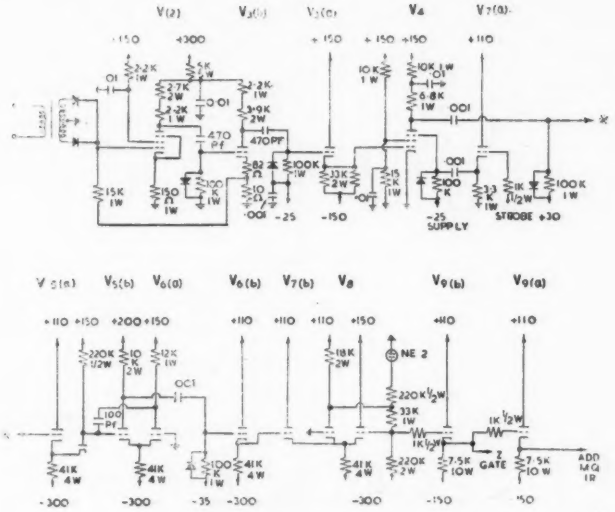


FIG. 9. Memory X and Y Read Write Drivers.

driver and the driving current flows through the selected pulse transformer. The magnitude of the driving current is almost independent of the variation in the driving tube characteristics. Minor variations in the currents through the individual transformers can be obtained by means of the individual cathode potentiometers of 100 ohms shown in the figure.

The pulse transformers are wound on C cores obtained from the English Electric Company, London. The primary turns for read and write are 125 each. They are wound on each leg of the C core and the secondary turns of 20 is wound 10 on each leg in series.

Figure 10 shows the circuit diagram of the pulse amplifier for the memory output. The rectifier diodes are biased slightly positive through the potential division at the cathode of the second stage of the amplifier. This helps in getting better signal to noise ratio. After the strobing at the 6AS6 ' And '



Valves : 2→6AH6; 3→12AT7; 4→6AS6;
5, 6, 7→6BQ7A; 8→6J6; 9→5687; All diodes 1N191

FIG. 10. Amplifier for Memory Output.

gate, the signal is fed to a univibrator to get a pulse of 4 μ sec. for reliable gating to the memory register.

SUMMARY

The T.I.F.R. computer is a parallel, binary, asynchronous single address machine. The arithmetic unit consists of a Memory Register, two (double rank) shift registers, the Accumulator and the M.Q. Register respectively, an Adder and a pair of True/Complement gates. A carry bypass logic is described which reduces the maximum carry delay time by a factor of $\frac{1}{3}$. The maximum carry delay time for 40 bits is 3 μ sec. The clear and gate drivers for the Registers and the driver (+ 40 V/−30 V) for True/Complement gating are of self-regulating type and their design and operation are dealt with in detail. The Add-Shift sequencing unit is described as a typical example of the control logic design. Other sequencing units perform their micro operations in exactly similar fashion. Schematic diagram of the fast access ferrite

core memory (access time 15 μ sec.) and some of the associated circuits are described.

ACKNOWLEDGMENTS

The authors are thankful to Dr. D. Y. Phadke for his guidance and encouragement, to Dr. R. Narasimhan for his overall guidance and help in the preparation of the paper, to Mr. P. V. S. Rao for the adder design and to Messrs. M. M. Farooqui, K. Bakhru, M. M. Dosabhai, B. B. Kalia, C. V. Srinivasan, T. R. N. Rao, V. M. Vengurlekar, R. R. Nargundkar and Miss V. K. Joglekar for their help in the various stages of the design, fabrication and testing of the circuits described in this paper and to Mr. V. S. Patil for preparing the drawings.

REFERENCES

1. Narasimhan, R. .. "On the system and engineering design of the digital computer at T.I.F.R.," To appear in the *Proc. Ind. Acad. Sci.*
2. Rao, P. V. S. and Basu, B. K. "A fast logical adder," (*Technical Report*).
3. Basu, B. K. and Rao, P. V. S. "A modified gating logic to improve the speed of operation of double rank counters," *Proc. Ind. Acad. Sci.*, 1957, **46**, 354–59.
4. Chu, J. C. .. "The oak ridge automatic computer," *Proc. Assoc. Comput. Machinery*, September 1952.
5. Papian, W. N. .. "The M.I.T. Magnetic-core Memory," *Proc. E.J.C.C.*, December 1953, 37–42.

1-61 Printed at The Bangalore Press, Bangalore City, by T. K. Balakrishnan, Superintendent, and Published by The Indian Academy of Sciences, Bangalore.

NOTICE TO AUTHORS

Scientific papers intended for publication in the *Proceedings of the Indian Academy of Sciences* can be accepted only when they are communicated by a Fellow of the Academy whose duty shall be to satisfy himself that such communications are fit to be read at the Meeting of the Academy and published in its *Proceedings*.

Papers should not ordinarily exceed fifty pages of foolscap. MSS. should be either typewritten or written in legible hand on one side of the paper. All papers should be carefully revised by the authors and should be absolutely in final form for printing. Position for text-figures should be indicated. Each paper shall conclude with a critical summary not exceeding 350 words.

Drawings, diagrams or other illustrations should be made on larger scale (preferably) twice the size than the ones in which they are intended to appear. They should be done in Indian ink on bristol board with lettering in pencil. Scale of magnification of camera lucida tracings should be indicated by the side of drawings. In certain special cases arrangements will also be made for monochrome lithographic and other colour plates. Reduction of illustrations desired should be indicated in pencil. Appropriate legends should accompany all drawings. Names of authors are to be marked in pencil on the left-hand corner of drawing sheets. Photomicrographs should be securely mounted with colourless paste.

All tables, quotations and footnotes which will be set hereafter (beginning from Vol. I, No. 2) in types smaller than the text, should be typewritten on separate sheets and placed with the text in proper sequence. Footnotes should be numbered in Arabic numerals.

References to literature in the text should be given, whenever possible, in chronological order, only the names of authors and years of publication, in brackets, being given. They should be cited in full after the summary, the authors' names following in alphabetical order. Thus,

Name or Names of author; Name of Journal (abbreviation) with a single underline; Year of publication; Number of Volume with a double underline, and lastly page. The following would be a useful illustration:—

Bergmann and Stather . . . . . Z. Physiol. Chem., 1926, 152, 189.

Two copies of slip-proof and wherever possible, a page proof for final revision will be sent to authors. All corrections are best made on the slip-proof which should be transmitted to the Office of the Academy. All proof corrections involve heavy expenses which would be negligible if the papers are carefully revised by the authors before submission.

Fifty free reprints including plates and with cover will be supplied for each paper. Additional copies can be supplied at cost on previous intimation.

Blocks appearing in the *Proceedings* will be available for purchase by their respective authors. Orders for the same should be sent along with the corrected proofs and in any case not later than one month after the date of publication of the paper. The price charged would be 25% of the actual cost of the blocks *plus* freight and despatching charges. If the blocks are reproduced in other journals or publications, due acknowledgment should be made in them to the *Proceedings*.

The original drawings and plates of blocks appearing in the *Proceedings* will be returned to such of the authors as may require them provided the cost of despatching such originals is borne by them.

CONTENTS

PAGE

Some Remarks on the Structure of Elementary Particle Interactions . . . .
Alladi Ramakrishnan, A. P. Balachandran and N. R. Ranganathan 1

On Compressible Boundary Layer on a Flat Plate with Uniform Suction . .
. . . . . A. C. Jain 12

Kinetic Studies in Ester Hydrolysis. Part X. The Alkaline Hydrolysis of Some Benzoic Esters in Mixed Solvents . . . . . .
. . . . . S. V. Anantakrishnan and P. S. Radhakrishnamurti 28

Some Theorems on Self-Reciprocal Functions . . . . . S. Masood 35

On the Logical and Circuit Design of the Digital Computer at T.I.F.R., Bombay
. . . . . B. K. Basu and S. P. Srivastava 44